UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2017

COURSE NO: CE 275
COURSE NAME: **DISCRETE MATHEMATICS**
CLASS: **CE II** TIME: 3 HOURS

Name: ____ Index Number: _______________

INSTRUCTIONS: During this exams, any communication with any person (other than the instructor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

SECTION (ONE)

The scoring for this exam is determined by the formula $\left[C - \left(0.25\times I\right)\right]\times 1.4$ ***where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. Identify the choice that best completes the statement or answers the questions.***

SECTION B

Instruction: Answer **ONLY TWO** questions from this section

Q1(a). Define and give an example each of the following:

(i) Equivalence Relation (ii). Partition of a set A

(iii). Cardinality of a set P

(b) (i). Assuming x as an arbitrary element, prove that the complement of the union of two sets A and B is the intersection of their complemets.

(ii). Prove the identity $\left(A\cup B\right)\cap C=\left(A\cap C\right)\cup\left(B\cap C\right)$ by using the membership table.

(c). Find the expansion of $\left(1+b\right)^{n}$ and hence show that

$$\frac{1}{\sqrt[5]{\left(32-x\right)}}=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1.6.11.\cdots.\left(5n-4\right)}{5^{n}.2^{5n+1}.n!}$$

Q2(a). The transitive closure R^{*} of a relation R is given as a set of ordered pair

$R^* = \{(a,b),(a,c),(a,d),(b,b),(b,c),(b,d),(c,b),(c,c),(c,d),(d,b),(d,c),(d,d)\}$

(i). Determine a matrix M_{R^*} representing the transitive closure R^* of the relation R.

(ii). Draw a digraph for the representation of the transitive closure R^* of the relation R

(iii). Is the transitive closure R^* of the relation R an equivalence relation?. Give examples to support your answers at each instance.

(b). Let $A = \{1, 2, 3, 4\}$ and R is a relation defied by "a divides b". Write R as a set of ordered pair, draw the digraph and find R^{-1}.

(c). Prove by mathematical induction that $5^n - 1$ is divisible by 4 for every positive integer $n \in R$.

Q3(a). Define and give an example each of the following:

(i). The Cartesian product of the sets A and B.

(ii) The relative complement of a set B with respect to a set A.

(b). Prove that if

(i) $a \mid b$ and $b \mid c$ then $a \mid c$ (ii). $a \mid b$ and $a \mid c$ then $a \mid (b+c)$

(c). Prove by mathematical induction that $n^3 - n$ is divisible by 3 for every positive integer n.

(d). Given that the Newton's Binomial expansion $(1+x)^{27} = \sum_{k=0}^{27} \binom{27}{k} x^k$, use this to compute the following expressions:

(i) $\sum_{k=0}^{27} \binom{27}{k} (-3)^{2k+1}$ **(ii).** $\sum_{k=0}^{27} k \binom{27}{k} (3)^{2k}$

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