COURSE NAME: THEORITICAL MECHANICS

COUSRE CODE: MA 278

1. Define Angular momentum. State and explain the Law of Conservation of Angular Momentum

ANSWER

Angular momentum: Can be defined as a vector quantity describing an object in circular motion; its magnitude is equal to the momentum of the particle, and the direction is perpendicular to the plane of its circular motion. Thus;

The Law of Conservation of Angular Momentum:

The law of conservation of angular momentum states that: When no external torque acts on an object, no change of angular momentum will occur.

Mathematically;

Suppose

Turque $(\tau) = 0$

$$\Rightarrow \frac{dL}{dt} = 0$$

$$\Rightarrow L = I\omega = constant$$

Then $\mathrm{L} = \text{constant}$

2. A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (Fig. 2). Initially, the mass revolves with a speed of v_1 = 2.4m/s in a circle of radius R_1 = 0.80m. the string is then pulled slowly through the hole so that the radius is reduced to R_2= 0.48m. What is the speed, v_2, of the mass now?

Fig 2

ANSWER

When speed: v_1 = 2.4m/s;

v_2 =?

Also when Radius: R_1 = 0.80m, and

R_2= 0.48m,

For small mass: $I = mr^2$

Conservation of Angular Momentum: $I\omega = I_0\omega_0$

$\Rightarrow mr^2\omega = mr_0{}^2\omega_0$

$\Rightarrow r_2\omega = r_{02}\omega_0$

But $v = r\omega$

$$\Rightarrow \omega = \frac{v}{r}$$

$$\Rightarrow \frac{r_2 v}{r} = \frac{r_{02} v_0}{r_0}$$

$\Rightarrow rv = r_0 v_0$

Also if

$v_0 = v_1 = 2.4\ m/s\ v$

$= v_2 = ?$

$r_0 = R_1 = 0.80\ m$

$r = R_2 = 0.48\ m$

$$\Rightarrow v = \frac{r_0 v_0}{r}$$

$$\therefore v = v_2 = \frac{r_0\ _0 v}{r}$$

$$\Rightarrow v_2 = \frac{80 \times 2.4}{0.48}\ 0.$$

$\Rightarrow v_2 = 4.0\ m/s$

$\therefore$ the speed, v_2, of the mass is $4.0\ m/s$

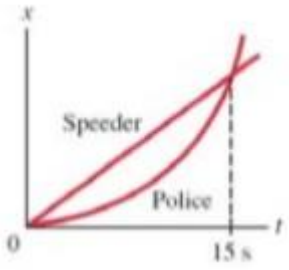
3. A car speeding at 150km/h passes a still police car which immediately takes off in hot pursuit. Using simple assumptions, such as that the speeder continues at constant speed, estimate how long it takes the police car to overtake the speeder. Then estimate the police car's speed at that moment and decide if the assumptions were reasonable

ANSWER

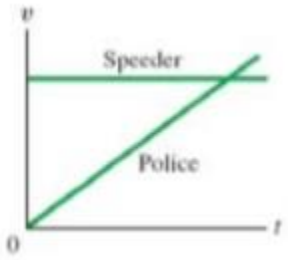
Consider the diagram below to make our assumption

Let make three simple assumption

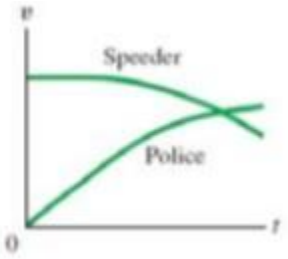
a)



b)



c)



Speeding car: velocity $(v) = {}^{150}\_\_\_\_\_^{\times 1000m}$; $1km = 1000m$

$and\ 1hour = 3600s$

$3600\ s$

$$\Rightarrow v = \frac{150000m}{3600\ s} = 41.7m/s$$

But distance (x) = velocity× time

$\Rightarrow x = vt = 41.7t$……..(1)

Police car:

Can go from 0 to 100 km/hr. = $\frac{10\times1000m}{3600\ s}$ $\frac{100000m}{3600\ s} == 27.8\ m/s$ in 5s

Also,

Constant acceleration = $a = \frac{\Delta v}{\Delta t} = \frac{27.8}{5} = 5.56\ m/s^2$

Using $x = x_0 + v_0t + \frac{1}{2}at^2$ $where\ (x_0 = 0, v_0 = 0)$

$$\Rightarrow x = 0 + (0)t + \frac{1}{2}(5.56)t^2$$

$\Rightarrow x = 2.78t^2$……..(2)

Equating equation 1 and 2

$41.7t = 2.78t^2$

$\Rightarrow 41.7 = 2.78t$

$$\Rightarrow t = \frac{41.7}{2.78}$$

$\therefore t = 15\ s \therefore$ it took the police car 15 s to overtake

the speeder.

The police car's speed:

$v = v_0 + at$

$\Rightarrow v = 0 + 5.56 \times 15 = 83.4\ m/s = 300\ km/hr$

$\therefore$ the police car's speed at that moment was $300\ km/hr$

This is clearly not possible. The acceleration of the police car would not be constant. The motor would not be able to maintain a constant torque and air resistance would increase with speed (roughly as the square of the speed). Also, the speeding car may have slowed down during the pursuit. The graph c may be a better representation of the chase but we do not have sufficient information do determine this.