UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, SEPT/OCT2022

COURSE NO: MA 380

COURSE NAME: STATISTICAL INFERENCE

CLASS: MA III

TIME: 2HRS:30MINUTES

Name: ____________________ Index Number: ____________

Answer any three questions

Question 1

a. Let $x_1 \ldots, x_n$ be a random sample from a normal distribution with mean μ and variances σ^2. Find the Maximum Likelihood Estimators of μ and σ^2

b. A sample of size four from a gamma distribution with p.d.f

$$f(x) = \frac{\beta^\alpha}{\Gamma_\alpha} x^{\alpha-1} e^{-\beta x} \quad ; \quad x > 0$$
$$0 \; ; \text{ elsewhere}$$

yielded x: 2,1,4,3 use the method of moments to estimate α and β

Question 2

a. Define unbiasedness

b. Show that $s^2 = \frac{1}{n-1}\sum(x - \bar{x})^2$ is unbiased for the population variance σ^2

$$\frac{\bar{X}^2}{\frac{1}{n}\Sigma x + \bar{X}^2}$$

Question 3

a. Define efficiency — Small var more eff

b. Define sufficiency

c. Let $x_1, \ldots x_n$ be a random sample from a normal distribution with mean μ and variance σ^2, show that $\bar{x} = \frac{\sum x}{n}$ is sufficient for μ.

$X - M)$

Question 4

a. State and prove Cramer Rao inequality

b. Let $x_1, \ldots x_n$ be a random sample from $N(\mu, \sigma^2)$, where μ and σ^2, are both unknown. Find a 95% confidence interval for . σ^2 if $\sum(x-\bar{x})^2 = 120$ and $n = 20$

Question 5

a. State Neyman-Pearson Lemma

b. Let $x_1 \ldots x_n$ be a random sample from $N(\mu, 64)$

(i) Find the best critical region C for testing $H_o: \mu = 80$ versus $H_1: \mu = 84$.

(ii) Find the sample size n and the critical values c so that $\alpha = 0.05$ and $\beta = 0.01$.

$(x-\mu)^2 \; \delta^{-2}$

$(\bar{x}-\mu) \cdot -2\delta^{-3}$

Examiner: Prof O. A. Fasoranbaku

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