# UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

**COURSE NO:** MN / MR / MC / El / CE 363

**COURSE NAME:** NUMERICAL ANALYSIS

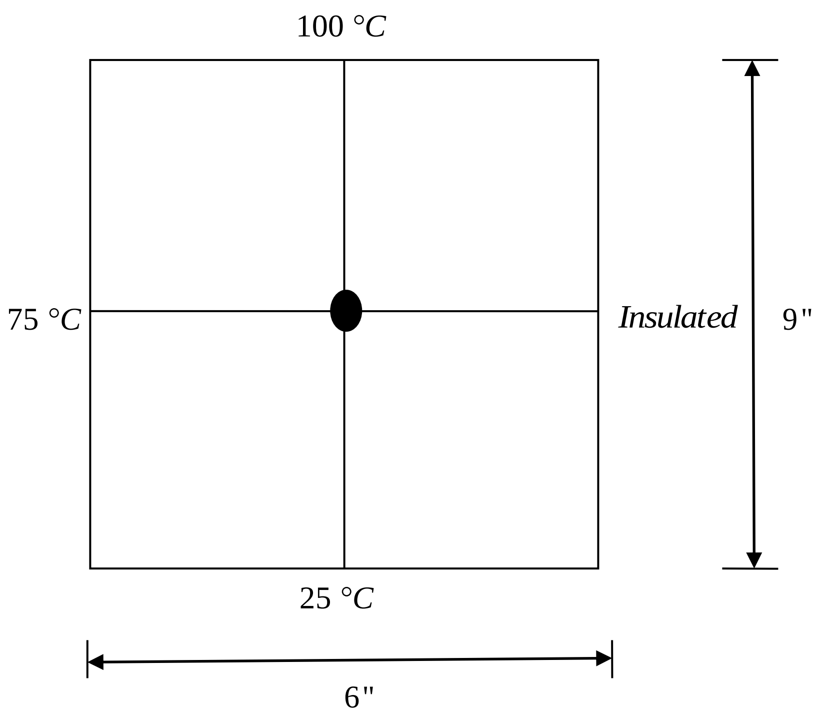
**CLASS:** MN /MR / MC / EL / CE III **TIME:** 3 HOURS

## SECTION A

(Answer **all** Questions in this section)

**Circle** the appropriate answer on the sheet

1. Find the steady-state temperature at the interior node as given in the following figure




(A) $53.57\,°C$ (C) $68.20\,°C$

(B) $66.40\,°C$ (D) $69.59\,°C$

2. The Newton-Raphson's algorithm for finding the cube root of $N$ is

(A) $x_{k+1} = \frac{1}{2}\left(x_k + \frac{N}{x_k}\right)$ (C) $x_{k+1} = \left(x_k + \frac{N}{x_k}\right)$

(B) $x_{k+1} = \frac{1}{3}\left(2x_k - \frac{N}{x_k^{\,2}}\right)$ (D) $x_{k+1} = \frac{1}{3}\left(2x_k + \frac{N}{x_k^{\,2}}\right)$

3. In which of the following method, we approximate the curve of solution by the tangent in each interval.

(A) Picard's method (C) Newton's method

(B) Euler's method (D) Runge-Kutta method

4. Given $3\frac{dy}{dx}+5y^2=\sin x,\ y(0.3)=5$ and using a step size of $h=0.3$, the value of $y(0.9)$ using Euler's method is most nearly

(A) -35.318 (B) $-35.318$ (C) $-658.91$ (D) $-669.05$

5. Which of the following method's convergence is sensitive to its starting value?
   (A) False position
   (B) Gauss-Seidel method
   (C) Newton-Raphson method
   (D) All of these

6. In the geometrical meaning of Euler's algorithm, the curve is approximated as a
   (A) Straight line
   (B) Circle
   (C) Parabola
   (D) Ellipse

7. Match the following:

| | |
|---|---|
| A. Newton-Raphson | 1. Integration |
| B. Runge-Kutta | 2. Root finding |
| C. Gauss-Seidel | 3. Ordinary Differential Equation |
| D. Simpson's Rule | 4. Solution of system of Linear Equations |

   (A) A2-B3-C4-D1
   (B) A3-B2-C1-D4
   (C) A1-B4-C2-D3
   (D) A4-B1-C2-D3

8. Solving an engineering problem requires four steps. In order of sequence, the four steps are
   (A) formulate, solve, interpret, implement
   (B) solve, formulate, interpret, implement
   (C) formulate, solve, implement, interpret
   (D) formulate, implement, solve, interpret

9. The velocity (m/s) of a body is given as a function of time (seconds) by $v(t)=200\ln(1+t)-t,\ t\geq 0$

   Using Euler's method with a step size of 5 seconds, the distance in meters traveled by the body from $t=2$ to $t=12$ seconds is most nearly
   (A) 3133.1
   (B) 3939.7
   (C) 5638.0
   (D) 39397

10. The root of the equation $f(x)=0$ is found by using the Newton-Raphson method. The initial estimate of the root is $x_0=3$, $f(3)=5$. The angle the line tangent to the function $f(x)$ makes at $x=3$ is $57^o$ with respect to the $x-$ axis. The next estimate of the root, $x_1$ most nearly is
   (A) $-3.2470$
   (B) $-0.2470$
   (C) 3.2470
   (D) 6.2470

11. The following data is given for the velocity of the rocket as a function of time. To find the velocity at $t=21$ s, you are asked to use the quadratic polynomial $v(t)=at^2+bt+c$ to approximate the velocity profile.

| $t$ | $(s)$ | 0 | 14 | 15 | 20 | 30 | 35 |
|---|---|---|---|---|---|---|---|
| $v(t)$ | $m/s$ | 0 | 227.04 | 362.78 | 517.35 | 602.97 | 901.67 |

The correct set of equations that will find $a$, $b$ and $c$ are

(A) $\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$ (B) $\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$ (D) $\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$

12. A homicide victim is found at 6:00 PM in an office building that is maintained at 73$^o$ F. When the victim was found, his body temperature was 85$^o$ F. Three hours later at 9:00 PM, his body temperature was recorded at 78$^o$ F. Assume the temperature of the body at the time of death is the normal human body temperature of 98.6$^o$ F. The governing equation for the temperature $\theta$ of the body is

$$\frac{d\theta}{dt} = -k(\theta - \theta_a)$$

where

$\theta =$ temperature of body, $^0$F
$\theta_a =$ ambient temperature, $^0$F
$t =$ Time in hours
$k =$ constant based on thermal properties of the body and air.

The estimated time of death most nearly is

(A) 2:11 PM
(B) 3:13 PM
(C) 4:34 PM
(D) 5:12 PM

13. Starting with $y_0 = 2$, the value of $y_4$ using the iteration formula $y_{n+1} = \sqrt{\dfrac{3(1+y_n)}{y_n}}$

(A) 2.104255
(B) 2.103731
(C) 2.130731
(D) 2.103713

14. In Runge-Kutta fourth order method, $k_4$ is

(A) $f(x_1 + \text{h}, y_1 + k)$
(B) $f(x_1 + h, y_1 + k_1)$
(C) $f(x_1 + h, y_1 + k_2)$
(D) $f(x_1 + h, y_1 + k_3)$

15. For $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \\ -2 \end{bmatrix}$ and using $[x_1\ x_2\ x_3] = [1\ 2\ 1]$ as the initial guess, the values of $[x_1\ x_2\ x_3]$ are found at each of the iteration as

| Iteration # | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 1 | 0.41666 | 1.116 | 0.96818 |
| 2 | 0.93989 | 1.0183 | 1.0007 |
| 3 | 0.98908 | 1.0020 | 0.99930 |
| 4 | 0.99898 | 1.0003 | 1.0000 |

At what first iteration number would you trust at least 1 significant digit in your solution?

(A) 1 (B) 2 (C) 3 (D) 4

16. The partial differential equation $\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = 0$ is classified as

(A) Elliptic
(B) Parabolic
(C) Hyperbolic
(D) None of the above

17. The truncation error in calculating $f'(2)$ for $f(x) = x^2$ by $f'(x) \approx \frac{f(x+h) - f(x)}{h}$ with $h = 0.2$ is

(A) -0.2
(B) 0.2
(C) 4.0
(D) 4.2

18. The following gas stations were cited for irregular dispensation by the Department of Energy. Which one cheated the most?

| Station | Actual gasoline dispensed | Gasoline reading at pump |
|---|---|---|
| Shell | 9.90 | 10.00 |
| Total | 19.90 | 20.00 |
| Star | 29.80 | 30.00 |
| Goil | 29.90 | 30.00 |

(A) Shell
(B) Total
(C) Star
(D) Goil

19. In a general second order linear partial differential equation with two independent variables,

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x \partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

where $A$, $B$, $C$ are functions of $x$ and $y$, and $D$ is a function of $x$, $y$, $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$, then the PDE is elliptic if

(A) $B^2 - 4AC < 0$
(B) $B^2 - 4AC > 0$
(C) $B^2 - 4AC = 0$
(D) $B^2 - 4AC \neq 0$

20. The Runge-Kutta 2nd order method can be derived by using the first three terms of the Taylor Series of writing the value of $y_{i+1}$ in terms of $y_i$ and all the derivatives of $y$ at $x_i$. If $h = x_{i+1} - x_i$, the explicit $y_{i+1}$ if the first three terms of the Taylor series are chosen for solving the ODE

$$\frac{dy}{dx} + 5y = 3e^{-2x}, y(0) = 7$$

(A) $y_{i+1} = y_i + (3e^{-3x_i} - 5y_1)h + 5\frac{h^2}{2}$

(B) $y_{i+1} = y_i + (3e^{-2x_i} - 5y_1)h + (-21e^{-2x_i} + 25y_i)\frac{h^2}{2}$

(C) $y_{i+1} = y_i + (3e^{-2x_i} - 5y_1)h + (-6e^{-2x_i})\frac{h^2}{2}$

(D) $y_{i+1} = y_i + (3e^{-2x_i} - 5y_1)h + (-6e^{-2x_i} + 5)\frac{h^2}{2}$

## SECTION B

Answer **two** Questions **only** in this section

1. a) The upward velocity of a rocket is given at three different times in the following table

| Time, t (s) | Velocity, v (m/s) |
|---|---|
| 5 | 106.8 |
| 8 | 177.2 |
| 12 | 279.2 |

Estimate the velocity at 10 s by

i) Linear interpolation for the last two data pairs and

ii) Quadratic interpolation by considering the table above

b) Consider $\frac{d^3y}{dt^3}+2\frac{d^2y}{dt^2}-\frac{dy}{dt}-2y=0;\ y(0)=1,\ \frac{dy}{dt}(0)=0, \frac{d^2y}{dt^2}(0)=-1$. Find by using Euler's iterative scheme

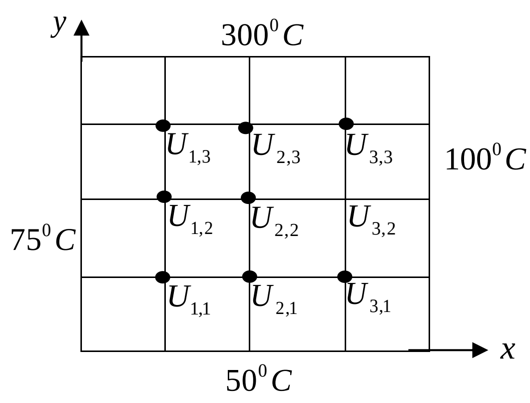
i) $y(1)$

ii) $\frac{dy}{dt}(1)$

iii) $\frac{d^2y}{dt^2}(1)$ using a step-size of 0.25

Round-off to 5 d. p.

2. A plate of $2.4m\times3.0m$ is subjected to temperature as shown below. Use the Gauss-Seidel with



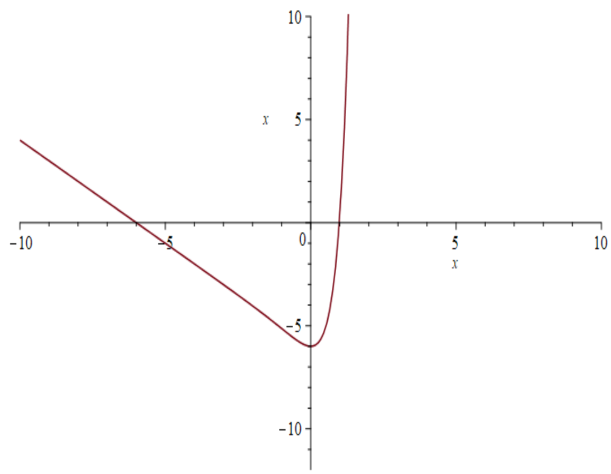

Successive Over relaxation method with a weighting factor of 1.4 to find the temperature at interior nodes. Conduct two iterations at all interior nodes. Assume the initial temperature to be $0^0C$.

3. a) i) Determine whether the Gauss-Seidel will converge for the system of equations

$$5x_1 - x_2 + x_3 - x_4 + x5 = 1$$
$$2x_1 - x_2 - x3 + x_4 - 6x_5 = 1$$
$$2x_1 - x_2 + 6x_3 - x_4 - x_5 = 1$$
$$-x_1 + 8x_2 + 2x_3 + x_4 - 2x5 = -1$$
$$x_1 + 2x_2 + 3x_3 - 9x_4 - 2x_5 = -1$$

ii) Apply the Gauss-Seidel iterative scheme to solve the system to 2 decimal places if the method converges.

b) Find all roots of the equation $xe^{2x} = x + 6$ correct to four decimal places if the graph of the equation is given as

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