UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

COURSE NO: MA 475
COURSE NAME: MATHEMATICAL ECONOMICS I
CLASS: MA IV **TIME:** 3 HOURS

Name**:** _____Index Number: _____

SECTION A OBJECTIVES

Answer all questions in this section

Each question is followed by four options lettered A to D. Find out the correct option for each question and circle the letter for the option you have chosen on your question paper. Give only one answer to each question.

1. **Marginal utility is the ……..**
   a. satisfaction achieved when a consumer has had enough of a product.
   b. total satisfaction received from consuming a given number of units of a product.
   c. extra satisfaction received from consuming one more unit of a product.
   d. average satisfaction received from consuming a product.
2. **Opportunity cost is the ……..**
   a. alternative chosen
   b. total cost of inputs
   c. alternative forgone
   d. cost of the product
3. **Total utility ……**
   a. is negative when marginal utility is declining
   b. has a constant rate of increase as a person consumes more and more of a good.
   c. cannot decrease as a person consumes more and more of a good.
   d. is equal to the sum of the marginal utilities of all units consumed.
4. **The price elasticity of supply is the ……**
   a. ratio of the percentage change in quantity demanded to the percentage change in price
   b. responsiveness of revenue to a change in quantity
   c. ratio of the percentage change in quantity supplied to the percentage change in price.
   d. the response of revenue to a change in price.
5. **The number of hours of work which the available labour force offers in return for a reward is known as**
   a. supply of labour
   b. mobility of labour

c. demand for labour
d. division of labour

6. **In the formulation of mathematical model in Linear programming, which one of the following includes the major steps that must be taken into consideration?.**
   a. determination of system constraints, which are also a linear relationship of the decision variables that reflect the limited resources of the problem.
   b. the development of an objective function that is a linear relationship of the decision variables.
   c. identification of decision variables.
   d. all the above are true
7. **If, when you consume another piece of candy, your marginal utility is zero, then**
   a. you should consume less candy
   b. you want more candy.
   c. you have maximized your total utility from consuming candy.
   d. you have not yet reached the point of diminishing marginal utility
8. **In the firms' production process, marginal cost**
   a. remains unchanged throughout
   b. falls continuously
   c. falls and later rises
   d. rises and later falls
9. **Suppose your marginal utility from consuming the 3rd slice of cake is zero, then your total utility from consuming cake is**
   a. negative.
   b. maximized.
   c. increasing.
   d. decreasing.
10. **If total utility increases at a decreasing rate as a consumer consumes more coffee, then marginal utility must**
    a. be negative.
    b. remains constant.
    c. decrease.
    d. increase also.
11. **Imagine a budget constraint between "good Y" on the vertical axis and "good X" on the horizontal. How would that budget line be affected if both income and the price of both goods fell?.**
    a. the new budget line must be parallel to the old budget line
    b. the budget line would not shift.
    c. the budget line must become shallower.
    d. the new budget line will have the same slope as the original so long as the prices of both goods change in the same proportion.
12. **A change in consumers' income will cause**
    a. a change in quantity demanded
    b. a change in demand
    c. savings to remain the same
    d. reckless spending

Use this information to answer questions 13 to 16.

A consumer has the utility function $U(D_1, D_2) = D_1 D_2$ and an income m of \$24.00. Initially, the price P_{X_1} of good 1 was \$1.00 and the price P_{X_2} of good 2 was \$2.00. Then the price of good 2 rose to \$3.00 and the price of good 1 stayed at \$1.00. If D_1 and D_2 are the demands for good 1 and good 2 respectively,

13. **Which of the relations below is the consumer's demand function for good 1?**

a. $D_1(P_{X_1}, P_{X_2}, m) = \dfrac{m}{2P_{X_1}}$

b. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{2m}{3P_{X_1}}$

c. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{m}{2P_{X_2}}$

d. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{2m}{3P_{X_2}}$

14. **Which of the relations below is the consumer's demand function for good 2?**

a. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{m}{2P_{X_1}}$

b. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{2m}{3P_{X_1}}$

c. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{m}{2P_{X_2}}$

d. $D_2(P_{X_1}, P_{X_2}, m) = \dfrac{2m}{3P_{X_2}}$

15. **What is the equilibrium quantities of "good 1" and "good 2" that were purchased at the time when the prices were** \$1.00 and \$2.00 **respectively?.**

a. $D_1 = 6\ units, D_2 = 12\ units$

b. $D_1 = 12\ units, D_2 = 4\ units$

c. $D_1 = 12\ units, D_2 = 6\ units$

d. $D_1 = 6\ units, D_2 = 6\ units$

16. **What is the equilibrium quantities of "good 1" and "good 2" that were purchased after the price of "good 1" had remained at** \$1.00 **and that of "good 2" had increased from** \$2.00 **to** \$3.00**.**

a. $D_1 = 6\ units, D_2 = 12\ units$

b. $D_1 = 12\ units, D_2 = 4\ units$

c. $D_1 = 12\ units, D_2 = 6\ units$

d. $D_1 = 6\ units, D_2 = 6\ units$

Consider figure 1 below consisting of three panels A, B and C as shown. **Use the information on these three panel to answer questions 17 to 19.**

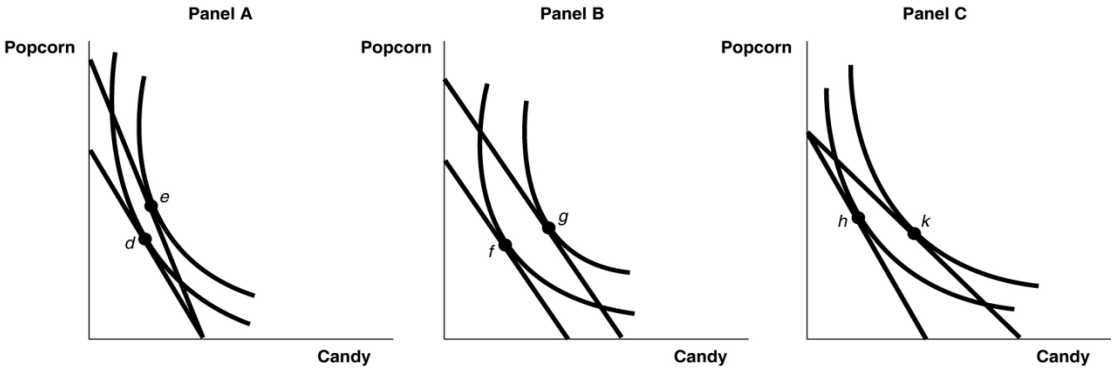

Figure 1

17. **With reference to figure 1 above, which of these panels show a change in income?**

a. Panel A.
c. Panel C.
b. Panel B.
d. none of the above panels

18. **With reference to figure 1 above, which of these panels show a change in the price of candy only?**

a. Panel A.
c. Panel C.
b. Panel B.
d. none of the above panels

19. **With reference to figure 1 above, which diagram demonstrates an increase in total utility following a decrease in the price of popcorn?**

a. the movement from d to e in Panel A
c. the movement from h to k in Panel C
b. the movement from f to g in Panel B
d. none of the above

20. **The slope of an indifference curve ……..**

a. is calculated by dividing the price of good on the vertical axis by price of the good on the horizontal axis.
b. is calculated by dividing the quantity of the good on the vertical axis by the quantity of good on the horizontal axis.
c. measures the marginal rate of substitution between the two goods in question.
d. measures total utility.

21. **What is the marginal rate of substitution?**

a. the rate at which the consumer is willing to trade one good for another so that she increases her utility.
b. the price ratio
c. the rate at which the consumer must give up one good to purchase additional unit of the other goods in the market.
d. the rate at which the consumer is willing to trade one good for another withot any loss in utility.

Use this information to answer questions 22 and 23.

A consumer spends \$360.00 per week on two goods X and Y. $P_X = \$3.00$ and $P_Y = \$2.00$. If his utility function is $U = 2X^2Y$.

22. **What quantity of good X does he buy each week in equilibrium?.**

a. 100 units
c. 60 units
b. 80 units
d. 40 units

23. **What quantity of good Y does he buy each week in equilibrium?.**

a. 20 units
c. 60 units
b. 40 units
d. 80 units

Use this information to answer questions 24 to 26.

The table below gives detailed information about Patience's utility from soup and sandwiches. The price of soup is $2.00 per cup and the price of a sandwich is $3.00. Patience has $18.00 to spend on these two goods.

Quantity of soup (cup)	Total utility	Quantity of sandwiches	Total utility
1	40	1	45
2	60	2	75
3	72	3	102
4	82	4	120
5	88	5	135
6	90	6	145

24. **What is Patience's marginal utility per dollar spent on the third cup of soup?.**

a. 72 units of utility
c. 12 units of utility
b. 36 units of utility
d. 6 units of utility

25. **If Patience maximizes her utility, how many units of each good should she buy?.**

a. 1 cup of soup and 5 sandwiches.
c. 4 cups of soup and 3.5 sandwiches.
b. 3 cups of soup and 4 sandwiches.
d. 6 cups of soup and 2 sandwiches.

26. **Suppose Patience's income increases from $18.00 to $23.00 but prices have not changed. What is her utility maximizing bundle now?.**

a. 4 cups of soup and 5 sandwiches.
c. 6 cups of soup and 5 sandwiches.
b. 5 cups of soup and 5 sandwiches.
d. 5 cups of soup and 4 sandwiches.

27. **The demand curve for a commodity is generally drawn on the assumption that ……**

a. technical knowledge remains constant
b. prices of factors of production do not fluctuate widely
c. taste, income and all other prices remain constant
d. The commodity has few substitutes.

28. **The law of diminishing marginal utility states that …….**

a. when the extra satisfaction from consuming a good becomes negative, total utility starts falling, other things constant.
b. eventually total utility falls as more of a good is consumed, other things constant.
c. the extra satisfaction from consuming a good decreases as more of a good is consumed, other things constant
d. the extra satisfaction from consuming a good increases slowly as more of a good is consumed, other things constant.

29. **If a 6% decrease in prices result in more than 6% decrease in quantity supplied, supply can be regarded as ……..**

a. perfectly inelastic
b. Unitary elastic

c. Elastic
d. perfectly elastic

30. **If Dawson prefers pizza to hamburgers and hamburgers to hot dogs, then if preferences are transitive, ……..**
    a. at times she could be indifferent among the three items.
    b. she could prefer hot dogs to pizza on some occasions but not hamburgers to hot dogs.
    c. she could prefer hamburgers to pizza on some occasions but not hot dogs to pizza.
    d. she must prefer pizza to hot dogs.

31. **The absolute value of the slope of the budget constraint is equal to**
    a. the price of good on the vertical axis divided by the price of the good on the horizontal axis.
    b. the marginal rate of substitution between the two goods in question.
    c. the price of good on the horizontal axis divided by the price of the good on the vertical axis
    d. quantity of the good on the vertical axis divided by the quantity of the good on the horizontal axis.

32. **The proportionate change in demand for commodity X divided by a proportionate change in the price of commodity Y is ……**
    a. cross elasticity of demand
    b. price elasticity of demand
    c. income elasticity of demand
    d. point of elasticity of demand

33. **If the price elasticity of supply of a commodity X is 0.60 and the price increases by 3 percent, then the quantity supplied of the commodity X will rise by ……..**
    a. 0.60 percent
    b. 0.20 percent
    c. 1.8 percent
    d. 18 percent

34. **Suppose we know that the price elasticity of supply of good X is equal to 1.2. Then, if its price will increase by 5%, we can predict with certainty that .....**
    a. the quantity supplied of that good will increase by 6%.
    b. the revenue of the firm producing that good will increase by 6%.
    c. the revenue of the firm producing that good will decrease by 6%.
    d. the quantity supplied of that good will decrease by 6%.

35. **If the cross-price elasticity between two commodities is -1.5,**
    a. the two goods are luxury goods.
    b. the two goods are complements.
    c. the two goods are substitutes.
    d. the two goods are normal goods.

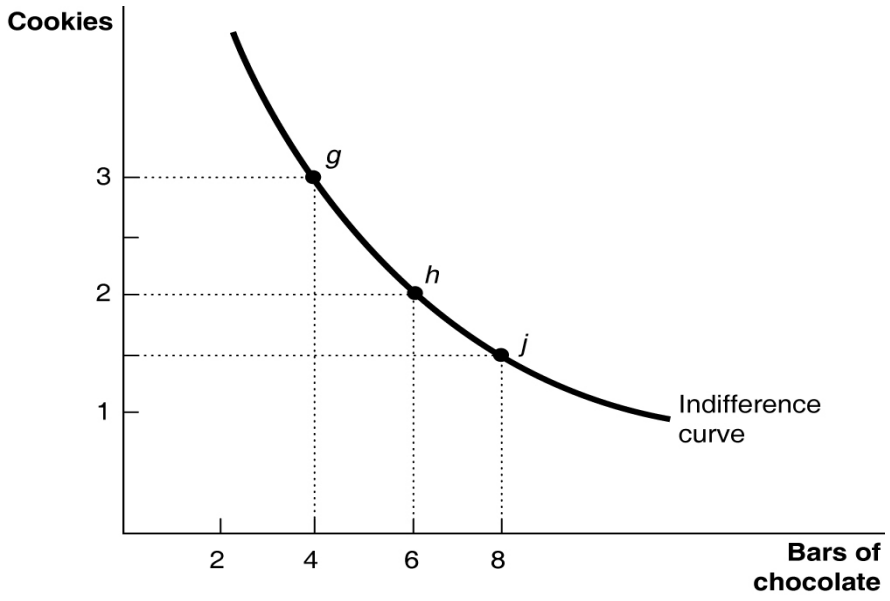

Figure 2

36. **Figure 2 above is an indifference curve of a rational consumer who derives her utility from consuming cookies and bars of chocolate. What is the marginal rate of substitution between *g* and *h*?**

a. $\frac{1}{3}$ cookie.

b. 3 cookies

c. $\frac{1}{2}$ cookie.

d. 2 cookies.

37. **The price elasticity of demand is ……**

a. the ratio of the percentage change in quantity demanded to the percentage change in price

b. the responsiveness of revenue to a change in quantity

c. the ratio of the change in quantity demanded to the change in price.

d. the response of revenue to a change in price.

38. **Which of the following statements is not true of indifference curves?.**

a. they are convex to the origin.

b. they are downward sloping.

c. they exhibit higher levels of utility as you move from the origin.

d. they could intersect.

Use this information to answer questions 39 to 42.

A consumer spends \$1000.00 every month on two goods X and Y. If P_X and P_Y are the prices of "good X" and "good Y" respectively with $P_X = \$10.00$ and $P_Y = \$20.00$. The utility function of the consumer is given by $U = XY$. As a result of market competition the price of good Y later changed from $\$20.00$ to $\$10.00$.

39. **What is the equation of the price line initially when the prices of "good X" and "good Y" were** $10.00 and $20.00 **respectively?.**

a. $2Y = -X + 100$

b. $2Y + 5X - 100 = 0$

c. $Y + X - 100 = 0$

d. $5Y + 2X - 100 = 0$

40. **What is the equation of the new price line after the price of "good X" had remained at** $10.00 **and that of "good Y" had decreased from** $20.00 **to** $10.00**.?**

a. $2Y = -X + 100$

b. $5Y + 2X - 100 = 0$

c. $Y + X - 100 = 0$

d. $2Y + 5X - 100 = 0$

41. **What is the equilibrium quantities of "good X" and "good Y" that were purchased every month at the time when the prices were** $10.00 and $20.00 **respectively?.**

a. $X = 25\ units, Y = 50\ units$

b. $X = 50\ units, Y = 50\ units$

c. $X = 50\ units, Y = 25\ units$

d. $X = 25\ units, Y = 25\ units$

42. **What is the equilibrium quantities of "good X" and "good Y" that were purchased every month after the price of "good X" had remained at** $10.00 **and that of "good Y" had decreased from** $20.00 **to** $10.00**?.**

a. $X = 25\ units, Y = 50\ units$

b. $X = 50\ units, Y = 50\ units$

c. $X = 75\ units, Y = 25\ units$

d. $X = 50\ units, Y = 25\ units$

Use this information to answer questions 43 to 47.

The table below gives detailed information in a world of two goods X and Y. If Q_X *and* Q_Y represent the quantities of good X and good Y respectively, P_X *and* P_Y represent the prices of good X and good Y respectively, TU_X *and* TU_Y represent the total utilities of good X and good Y respectively and MU_X *and* MU_Y represent the marginal utilities of good X and good Y respectively.

Q_X	TU_X	MU_X	$\frac{MU_X}{P_X}$	Q_Y	TU_Y	MU_Y	$\frac{MU_Y}{P_Y}$
1	30	30	15	1	50	50	5
2	39	9	4.5	2	105	55	5.5
3	45	6	3	3	148	43	4.3
4	50	5	2.5	4	178	30	3
5	54	4	2	5	198	20	2
6	56	2	1	6	213	15	1.5

43. **What are the prices of good X and good Y?.**

a. $P_X = \$2.00; \quad P_Y = \$10.00$

b. $P_X = \$1.00; \quad P_Y = \$5.00$

c. $P_X = \$10.00; \quad P_Y = \$2.00$

d. $P_X = \$5.00; \quad P_Y = \$1.00$

44. **How many potential equilibrium positions can be identified from the table under consideration?.**

a. 1.
c. 3.
b. 2.
d. 4.

45. **If the consumer's income is $60.00, which of the options below would the equilibrium position be considered optimum?.**
    a. equilibrium position containing 1 unit of good X and 2 units of good Y.
    b. equilibrium position containing 3 units of good X and 4 units of good Y.
    c. equilibrium position containing 5 units of good X and 5 units of good Y
    d. (B) and (C).

46. **If $60.00 is the total expenditure on a possible equilibrium position, which of the following could represent another total expenditure of an equilibrium position if any other(s) exist(s)?.**
    a. $46.00.
    c. $36.00
    b. $24.00.
    d. None of the above.

47. **If $60.00 is the total expenditure on the optimum equilibrium position, what is the total utility associated with this equilibrium position?.**
    a. 196
    c. 223.
    b. 204.
    d. 252.

48. **When total utility is constant, it means marginal utility is ……..**
    a. Increasing
    c. Decreasing
    b. one
    d. Zero

49. **The price elasticity of demand generally tends to be …….**
    a. smaller in the long run than in the short run.
    b. smaller in the short run than in the long run.
    c. larger in the short run than in the long run.
    d. unrelated to the length of time.

50. **Suppose we know that the price elasticity of demand of good X is equal to -1.2. Then, if its price will increase by 5%, we can predict with certainty that ……..**
    a. quantity demanded of that good will increase.
    b. the revenue of the firm producing that good will increase by 6%.
    c. the revenue of the firm producing that good will decrease by 6%.
    d. the quantity demanded of that good will decrease by 6%.

51. **A 10% increase in the price of movie ticket in AGC Club house leads to a 15% decrease in the number of tickets sold, indicating the demand for movie tickets in AGC Club house is…..**
    a. elastic.
    c. Unit elastic.
    b. inelastic.
    d. Can not tell from the information given.

52. **If the cross-price elasticity between two commodities is 1.5, ……**
    a. the two goods are luxury goods.
    c. the two goods are substitutes.
    b. the two goods are complements.
    d. the two goods are normal goods.

53. **If the number of degrees of freedom in the use of the simplex method for solving LP models is two and can be represented by any two variables which can be chosen to set to any arbitrary value. The simplex method uses zero for the arbitrary value. The values that are currently set to zero are called ……. and the others are called …… respectively.**

a. Basic variables, Non-basic variables
b. Non-basic variables, Basic variables
c. Basic solution, Basic feasible solution
d. Basic feasible solution, Basic solution.

54. **When the total product curve is falling, marginal product is always ……..**

a. Zero.
b. Negative.
c. Falling
d. Positive

55. **In perfect competition, the short–run supply curve is equal to the …….**

a. Average variable cost curve.
b. Marginal cost curve
c. Total cost curve
d. Average fixed cost curve.

56. **Which of the following cost of production does not change with output?.**

a. Marginal cost (MC)
b. Total Fixed Cost (TFC)
c. Average Fixed Cost (AFC)
d. Variable Cost (VC)

57. **Which of the following would not cause a shift in the demand curve?.**

a. The price of the commodity itself
b. Prices of other commodities
c. The income of consumers
d. Consumers taste of luxuries

Consider the Cobb-Douglass production function of a firm given by $N(x,y) = 20x^{1/2}y^{1/2}$. If x is the number of units of labour and the cost is \$40.00 per unit whilst y is the number of units of capital with the cost of \$120.00 per unit of capital. Assuming the levels of production, units of labour and capital are all to the nearest unit and the budget is \$300, 000.00. Use this information to answer questions 58 to 60.

58. **What is the number of units of labour required for optimal production?.**

a. 1250
b. 43302
c. 3750
d. 43301

59. **What is the number of units of capital required for optimal production?.**

a. 1250
b. 43302
c. 3750
d. 4330

What is the optimal production of the firm within this level of production?.

a. 1250
b. 43302
c. 3750
d. 4330

SECTION B

Instruction: Answer **TWO** questions only from this section. All questions carry equal marks

Q1(a). The table below shows the price and yearly quantity sold of souvenir T-shirts in the town of Crystal Lake according to the average income of the tourists visiting.

Price of T-shirt	Quantity of T-shirt demanded when the average tourist income is $20,000	Quantity of T-shirt demanded when the average tourist income is $30,000
$4	3,000	5,000
$5	2,400	4,200
$6	1,600	3,000
$7	800	1,800

(i). Using the midpoint method, calculate the price elasticity of demand when the price of a T-shirt rises from $5.00 to $6.00 and the average tourist income is $20,000. Similarly, calculate it when the average tourist income is $30,000.

(ii). Using the midpoint method, calculate the income elasicity of demand when the price of a T-shirt is $4.00 and the average tourist income increases from $20,000.00 to $30,000.00. Similarly, calculate it when the price is $7.00.

(b). An individual lives in a world where there are only two goods, X and Y. His utility function per period is given by

$$U(X,Y) = 50X - \frac{1}{2}X^2 + 100Y - Y^2$$

The price of X is Gh¢4.00 and his income per period is Gh¢672.00

(i). Derive his demand functions for X and Y

(ii).If the price of Yis Gh¢14.00, using your demand function for X and Y in (i) above, find the number of units of X and Y he buys in equilibrium?

Q2(a). Assume that a consumer's satisfaction depends on income and leisure. The consumer's utility function is given by

$$U = g(L, y)$$

where L denotes leisure and y is the income. If the income of the consumer depends on the amount of work performed by him/her and it is denoted by W, the wage rate by r and T is the total amount of available time. Show that

(i) $\dfrac{dU}{dW} = -g_1 + g_2 r$

(ii) $\dfrac{d^2U}{dW^2} = g_{11} - 2g_{12}r + g_{22}r^2$

(b). Given that the profit function for a firm producing two goods X and Y is

$$\pi = 160X - 3X^2 - 2XY - 2Y^2 + 120Y - 18$$

(i). Determine the level of inputs that maximizes profit

(ii). Test the second-order condition for maximization

(iii). Evaluate the function at the critical values of X and Y

Q3(a). A firm has the following long-run production function given by

$$X(A, B, C) = bA^{0.5}B^{0.5}C^{0.5}$$

Where X is weekly output, b is a positive constant, A, B and C are the weekly inputs of the three factors used. The price of A is Gh¢1.00, the price of B is Gh¢9.00 and the price of C is Gh¢8.00.

(i). Derive the following:

(α). The firm's long run total cost function

(β). The firm's long run average cost function

(ρ). The firm's long run marginal cost function

(ii). If in the short-run, factor C is fixed while factors A and B are variables derive the following:

(α). The firm's short run total cost function

(β). The firm's short run average cost function

(ρ). The firm's short run marginal cost function

(λ). The firm's short-run average variable cost function

(iii). Derive an equation in the form $C = f(X)$ showing the optimum quantity of the fixed factor C for the firm to acquire as a function of the intended output of X

(b). Write down the dual form of the following primal linear programming problems

(i) Maximize

$$z = 50x_1 + 100x_2$$

subject to:

$$2x_1 + 3x_2 \leq 60$$

$$4x_1 + 2x_2 \leq 10$$

$$2x_1 + 3x_2 \leq 50$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

(ii). Maximize

$$z = 45x_1 + 80x_2$$

Subject to:

$$5x_1 + 20x_2 \leq 400$$

$$10x_1 + 15x_2 \leq 450$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

Good Luck

***Examiner: P. K. Nyarko**/ E. Danso-Addo*