UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY 2018

COURSE NO: MA 274

COURSE NAME: **ORDINARY DIFFERENTIAL EQUATIONS**

CLASS: MA II **TIME:** 3 HOURS

Name**:** _____Index Number: _____

SECTION A: ANSWER ALL QUESTIONS. EACH QUESTION CARRIES 1 MARK

1. An integrating factor that makes the differential equation $ydx + (3 + 3x - y)dy = 0$ is……...

2. State whether the equation $x^2 + xy - y^2$ is homogeneous or not and write down the degree.……

3. Find the general solution of the differential equation $2xydx - (x^2 - y^2)dy = 0$

   ……

4. Find the general solution of the second order differential equation $y'' - y' - 6y = 0$

   …….

5. If the auxiliary equation of a homogeneous linear differential equation contains roots

$r = m$(twice), then the solution of the differential equation is ……

6. The singular points for $(x^2 + 1)y''' + y'' - x^2y = 0$ is ……..…..

7. According to Newton's second law of motion, the acceleration a of a body of mass m is proportional to the total force F acting on the body. The standard implementation of this relationship is ……

8. All possible solutions to a differential equation is called the ……to the equation.

9. How much (to the nearest dollar) must a 25-year-old woman invest now at 8% compound continuously to have $40,000 when she is 60?.....

10. An ordinary differential equation is separable if the differential equation is of the form ……

11. A 100-volt electromotive force is applied to an RC series circuit in which the resistance is 200 ohms and the capacitance is 10^{-4} farads. Find the charge $q(t)$ on the capacitor if $q\ (0) = 0$. The current $i(t)$…….

12. The equation $u' = -k(u - T)$ is an ……...because it has one, and only one, independent variable.

13. A ……consists of a large number of line segments, usually drawn on a rectangular grid.

14. The Laplace transform of the function $f(x) = tsin2t$ is ……...

15. The differential equation $\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^2 + 4y = 4\cos x$ is a 3rd order ordinary differential equation with degree ……..

16. The …….. helps us to know how we can tell whether some particular equation has a solution or not.

17. A function f is ……. at a point *a*, if it can be represented by a power series in $x - a$ with a positive radius of convergence.

18. For the linear-second order differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$, x_0 is ……. if $a_2(x_0) \neq 0$, $a_2(x)$ and $a_1(x)$ are analytic.

19. Classify the following differential equation: $e^x \frac{dy}{dx} + 3y = x^2y$

a. Separable and not linear
b. Linear and not separable
c. Both separable and linear
d. Neither separable nor linear

20. Classify the following differential equation: $w\frac{dw}{dt} + 3t = 10$

a. Separable and not linear
b. Linear and not separable
c. Both separable and linear
d. Neither separable nor linear

21. Classify the following differential equation: $\frac{dx}{dt} = \frac{x+2xt+cost}{1+t^2}$

a. Separable and not linear
b. Linear and not separable
c. Both separable and linear
d. Neither separable nor linear

22. Classify the following differential equation: $\frac{dz}{dt} = 1 + z + t + zt$

a. Separable and not linear
b. Linear and not separable
c. Both separable and linear
d. Neither separable nor linear

23. Suppose *y* is a function of *x*. Which of the following is the solution of $\frac{d(x^3y)}{dx}$?

a. $3x^2y + x^3\frac{dy}{dx}$ b. $3x^2y$ c. $3x^2\frac{dy}{dx}$ d. $3x^2y + x^3$

24. Identify the functions $p(t)$ and $q(t)$ if the differential equation $\frac{dx}{dt} = \frac{x+t^2-2x\sqrt{t}}{t}$ is written in the form $\frac{dx}{dt} + p(t)x = q(t)$.

a. It is not possible to write the equation in the form described
b. $p(t) = 2\sqrt{t} - 1, q(t) = t^2$
c. $p(t) = 1 - 2\sqrt{t}, q(t) = t$
d. $p(t) = \frac{2\sqrt{t}-1}{t}, q(t) = t$

25. An integrating factor, $I(x)$, is found for the linear differential equation $(1 + x^2)\frac{dy}{dx} + xy = 0$, and the equation is rewritten as $\frac{d}{dx}(I(x)y) = 0$. Which of the following options is correct?

a. $I(x) = \frac{e^{x^2}}{2}$

b. $I(x) = \sqrt{1 + x2}$

c. $I(x) = \sqrt{1 + x2}$

d. It is not possible to find such a function $I(x)$

26. A principal of $60 is invested. The value, $I(t)$, of investment, t days later satisfies the differential equation $\frac{dI}{dt} = 0.02I + 5$. Find the value of the investment after 27 days, correct to two decimal places.

a. $97.65 b. $176.34 c. $281.96 d. $301.38

27. Classify the following differential equation: $e^x \frac{dy}{dx} + 3y = x^2 y$

a. Separable and not linear
b. Linear and not separable
c. Both separable and linear
d. Neither separable nor linear

28. Consider the differential equation $y'' + 2y' - 8y = 0$. If the derivatives are taken with respect to t, determine the values of r for which the given differential equation has solution of the form $y = e^{rt}$.

a. r = 1 and r = 4 b. r = 1 and r = -4 c. r = -1 and r = 4 d. r = 2 and r = -4

29. Which of the following is an exact differential equation?

a. $(2x + 3y)dx + (3x + 2y)dy = C$
b. $(4x + 3y)dx + (x + 2y)dy = C$
c. $(2x + 3y)dx + (4x + 3y)dy = C$
d. $(2x + 3y)dx + (5x + y)dy = C$

30. The solution of the differential equation $3xy' + y = 12x$ is ……

31. The equation $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 15x = 0$ is a second order, second degree differential equation. True/False

32. If $y = f(x)$ is a given function, then the derivative df/dx cannot be interpreted as the rate of change of f with respect to x. True/False

33. The differential equation $\frac{d^3y}{dx^3} - \left(\frac{dy}{dx}\right)^2 + 4y = 4e^x \cos x$ is said to be non-linear. True/False

34. The differential equation y" = f (t, y, y') is said to be linear if it can be written in the standard form y" + p(t) y' + q(t)y = g(t), where the coefficient of y" is equal to 1. True/False

35. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 yr. After 75 yr the ratio of the population P to the initial population P_o is $\frac{5}{2}$. True/False

36. $\frac{1}{P(D)}[e^{rx}] \neq \frac{e^{rx}}{P(r)}, if\ P(r) \neq 0$. True/False

37. Newton's Law of Universal Gravitation, Maxwell's field equations, the motions of the planets, and the refraction of light are important physical examples which can be expressed using differential equations. True/False

38. Differential equations together with the initial condition forms an initial value problem. True/False

39. In Laplace transformation, one element in the time domain corresponds to many elements in the complex domain. True/False

40. Is the differential equation $(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$ exact? Yes/No

SECTION B: ATTEMPT ANY ONE (1) QUESTION

1a. Given that the non- homogeneous second order differential equations is of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = Q(x)$$

Show that the Complimentary Function is given by $a\frac{d^2k}{dx^2} + b\frac{dk}{dx} + ck = 0$, where a, b, and c are constants.

b. Find the general solution to $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 15x - 7$

2. The population of Tarkwa community is known to increase at a rate proportional to the number of people present at a time t. If the population has doubled in 6 years, how long will it take to triple?

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