UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC. 2018

COURSE NO: MA 273

COURSE NAME: REAL ANALYSIS

CLASS: **MA II** **TIME:** 2 HRS 30 MIN

Name: ____Index Number: _______________

SECTION A: Answer all Questions in this section

Q1. Determine whether the following statements are True T or False F, and circle appropriately.

(a) If S is a nonempty subset of the natural numbers N, then there exists an element $m \epsilon S$ such that $m \leq k$ for all $k \epsilon S$. **T** /**F**

(b) The open bounded interval (0,1) is an open set in R. **T** /**F**

(c) For any bounded sequence $\{a_n\}$, the sequence $m_n = \inf\{a_k : k \geq n\}$ is an increasing sequence **T** /**F**

(d) Every nonempty bounded subset of real numbers has a least upper bound in R **T** /**F**

(e) If m is a lower bound for the set S, and $m' < m$, then m' is not a lower bound for S. **T** /**F**

(f) The ordered field Q of rational is not Archimedean **T** /**F**

(g) All divergent sequences are unbounded. **T** /**F**

(h) The set $\{1, \ 1.1, 0.9, 1.01, 0.99, 1.001, 0.999, \ldots\}$ is not bounded **T** /**F**

(i) The set $\{-0.9, \ 0.9, -0.99, 0.99, -0.999, 0.999, \ldots\}$ has $g.l.b = 1$ and the $l.u.b = -1$ **T** /**F**

(j) A set S is called countable if and only if there is one and only one correspondence between S and the set of natural numbers, N. **T** /**F**

(k) The sequence $\{a_n\}_{n=1}^{\infty} = \{(-1)^n\}_{n=1}^{\infty}$ is a convergent sequence **T** /**F**

(l) The union of two uncountable sets is countable **T** /**F**

(m) If $I_n = \left[\frac{-1}{n}, \frac{1}{n}\right]$, and $\cap I_{n\geq 1}$ is an empty set, then I_n is a nested Interval **T** /**F**

(n) For $a \in \mathbb{R}, n \in N$, the sequence $\{a^n\}$ converges if and only if $|a| < 1$ **T** /**F**

(o) (t) All sequences in $\left(1, \frac{1}{3}\right)$ converge to a limit as $n \to \infty$ **T** /**F**

(p) The set of all limits points of a is called a derived set of A **T** /**F**

(q) The sequence $\left\{\sin \frac{n\pi}{2}\right\}, \forall \ n \in N$ is a convergent sequence. **T** /**F**

(r) If $x_0 \in (a, b)$ then for a small value $\varepsilon \epsilon \mathbb{R}$ $(x_0 - \varepsilon, x_0 + \varepsilon) \subset (a, b)$ **T** /**F**

(s) The empty set $\emptyset$ and the whole space X are closed sets. **T** / **F**

(t) Every singleton subset of a metric space is closed. **T** / **F**

SECTION B: Answer <u>Two</u> Questions from this section.

Q2. a. Prove that the set $S = \{0.\ 1,\ -1\}$ is a field.

b. Prove that if the sequence (x_n) converges to l and the sequence (y_n) converges to m as $n \to \infty$ then the sequence $(x_n + y_n)$ converges to $(l + m)$.

c. (i) State the Nested Interval lemma or theorem.

(ii) Show that the interval $I = \left[0, \frac{1}{n}\right]$ is nested.

Q3. (a) Define the following terms: (i) Subsequence (ii) Cauchy sequence

(b) Prove that every convergent sequence is a Cauchy sequence.

(c) Let $a_n \coloneqq \left(1 + \frac{1}{n}\right)^n, \forall$ integers $n \geq 1$; and $b_n \coloneqq \left(1 + \frac{1}{n}\right)^{n+1}, \forall$ integers $n \geq 1$

Prove that: (i) $a_n \leq b_n, \forall\, n \geq 1,$ (ii) $\{b_n\}$ is a monotone decreasing sequence

Q4. (a) Let the set R^2 of all ordered pairs of real numbers with metric space ρ_2 be defined by

$$\rho_2(x, y) = \left[\sum_{i=1}^{2} (x_i - y_i)^2\right]^{\frac{1}{2}}$$

where $x = (x_1, x_2)$ and $y = (y_1, y_2)$, for arbitrary $x, y \in R^2$. Describe and illustrate graphically, the sets (i). $B_1(0)$ (ii). $\bar{B}_1(0)$ and (iii). $S_1(0)$ where $0 = (0,0)$ in R^2.

(b) Let X be a metric space with metric d. Show that d defined by $d\,(x, y) = \sqrt{\sum_{k=1}^{n} (x_k - y_k)^2}$ where x and y are arbitrary elements of X, is also a metric on X.

***Examiners: J Acquah/** L. Brew*