UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

COURSE NO: MA 271

COURSE NAME: CALCULUS OF SEVERAL VARIABLES

CLASS: MA II **TIME:** 3 HOURS

Name**:** _____Index Number: _____

Answer <u>Three</u>** Questions in All. Questions **<u>One</u>** and Any Other **<u>Two</u>

Q1.(a) Determine the domain and the range of the function $f(x,y,z) = \sqrt{x^2+y^2+z^2-2}$.

(c) Evaluate $\lim_{(x,y)\to(0,0)} \left[\frac{(x-1)^2 lnx}{(x-1)^2+y^2}\right]$ and prove that the limit exists.

(d) Using the $\varepsilon - \delta$ definition of limit, prove that $\lim_{(x,y)\to(2,1)} (xy - 3x + 4) = 0$.

Q2.(a) If $u = f(x,y)$, where $x = e^s cost$ and $y = e^s sint$, show that

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = e^{-2s}\left[\left(\frac{\partial u}{\partial s}\right)^2 + \left(\frac{\partial u}{\partial t}\right)^2\right]$$

(b) If $z^3 - xz - y = 0$, prove that $\frac{\partial^2 z}{\partial x \partial y} = -\frac{3z^2+x}{(3z^2-x)^3}$

(c) Given that $(x,y) = yx^{\frac{2}{5}} + x\sqrt{y}$, estimate by a differential the change in f from (32, 16) to (35, 18)

Q3.(a) Given $\iint_R (x^2 - xy)\, dA$, where R is the region enclosed by the line $y = x$ and the curve

$y = 3x - x^2$,

i) Sketch the region enclosed by the double integral R.

ii) Evaluate the area enclosed by the double integral R.

(b) Find the volume V of the region $\Re^3$ defined by $V = \iiint_R dv$, which is bounded by the graphs of the curve $y^2 + z^2 = 9$ and the lines $x + z = 3$ and $x = 0$.

Q4.(a) Find the area enclosed by the elliptic $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Verify Green's theorem in the plane for $\oint_c [(x^3 - x^2y)\, dx + (xy^2)\, dy]$, where C is the boundary of the semi-circular region D on the upper half plane between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(c) Evaluate the surface integral $\iiint_s (x + z)\, dv$ where S is the region under the plane $x + 2y + 3z = 6$ in the first Octant.

***Examiners: J. Acquah**/ P. K. Nyarko*