UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATION, MAY 2018

COURSE NO: MA 270 Unihubgh.com

COURSE NAME: **DIFFERENTIAL AND INTEGRAL CALCULUS**

CLASS: MA II TIME: Three (3) Hours

Name: ____ Index Number: ________________

Instruction: Attempt **ANY Four (4)** Questions

1a. Define the improper integrals of, (i) the first kind and (ii) the Second kind.

1b. Classify, with reasons, the following integrals according to the kind (type) of improper integral,

(i) $\int_{-1}^{5}\frac{dx}{(x-1)^3}$, (ii) $\int_0^{\infty}\frac{e^{-x}}{\sqrt{x}}dx$, (iii) $\int_0^{1}\frac{\sin x}{x}dx$, (iv) $\int_{-\infty}^{\infty}\frac{x^3dx}{x^4+x^2+1}$, (v) $\int_0^{\frac{\pi}{2}}\frac{dx}{1-\sin x}$, (vi) $\int_0^{\frac{\pi}{2}}\frac{e^{-x}\cos x}{x}dx$,

(vii) $\int_1^{2}\frac{\ln x}{\sqrt[3]{8-x^3}}dx$, (viii) $\int_0^{\infty}\frac{e^{-x}dx}{\sqrt{x\ln(x+1)}}$, (ix) $\int_0^{\infty}\frac{dx}{\sqrt[3]{x^4+x^2}}$, (x) $\int_{-\infty}^{\infty}\left(\frac{2+\sin x}{x^2+1}\right)dx$

1c. Test for convergence or otherwise of the integral $\int_2^{\infty}\frac{x^3-1}{\sqrt{x^6+16}}dx$

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2. Evaluate the following improper integrals, (a) $\int_0^{9}\frac{dx}{(x-1)^{2/3}}$, (b) $\int_0^{\infty}\frac{x\,dx}{x^4+1}$

3a. Express $\int_0^2 x\left(\sqrt[3]{8-x^3}\right)dx$ as beta function and show that its value is equal to $\frac{16\pi}{9\sqrt{3}}$

3b. Express $\int_0^1\frac{dx}{\sqrt{-\ln x}}$ as gamma function and find its value.

3c. Evaluate $\Gamma\left(-\frac{5}{2}\right)$ where, $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

4a. Evaluate $L\left\{\frac{\sin at}{t}\right\}$

4b. If $L\{f\}$ and $L\{f'\}$ are defined, show that $L\{f'\} = -f(0) + sL\{f\}$, where, L{ } and $L^{-1}\{\}$ are the Laplace Transform and its inverse respectively.

4c. Given the Bessel's equation, $x^2y''(x) + xy'(x) + (n^2 - n^2) = 0$. Show that the Bessel's equation has regular singularity at x = 0.

5a. Prove that $\frac{d}{dx}\left(J_n(x)\right) = \frac{1}{2}\{J_{n-1}(x) - J_{n+1}(x)\}$, where, $J_n(x)$ is Bessel function of first kind and order n. 5b. Hence, show that $\frac{d}{dx}\left(J_0(x)\right) = -J_1(x)$.

6a. When do we say, a function f(x) is said to be period at period T. Give two examples.

6b. Determine the Fourier series to the periodic function, $f(x) = \frac{x}{2}$, $0 < x < 2\pi$, and f(x) = f(x+2π), i. e. Period = 2π.

6c. Define a transformation matrix with appropriate example. Hence, what is orthogonal transformation?

Examiner: Prof F. I. Alao