SECOND SEMESTER EXAMINATIONS MAY, 2018

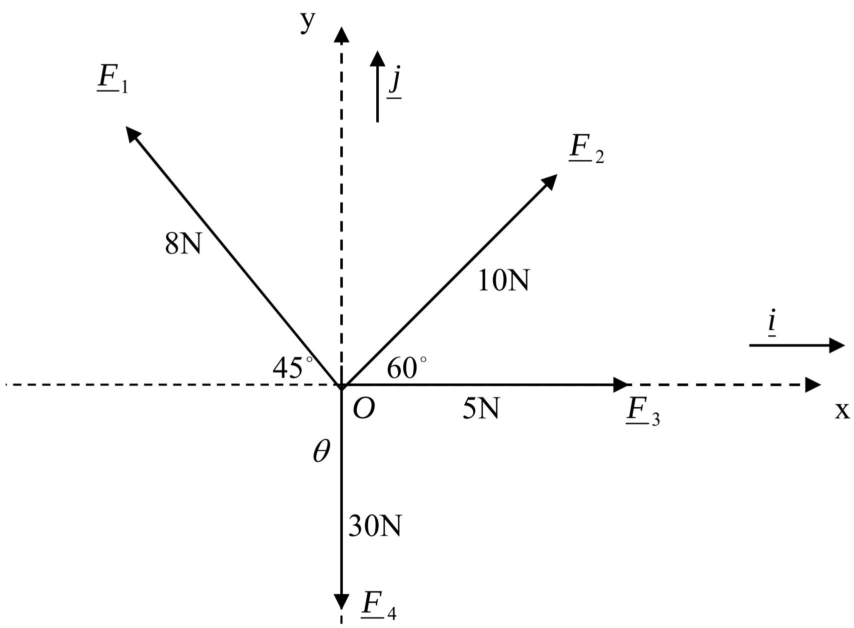
COURSE NUMBER: MA 178

COURSE NAME: VECTOR APPLICATIONS

CLASS: MA I TIME: 3 HOURS

Instruction: Answer any **three** questions. Each qustion must be on a fresh page.

Q1(a). A particle is acted on by forces of magnitude $8N$, $10N$, $5N$ and $30N$ in the directions as shown in the figure below: find the resultant of these forces and leave your answer in a surd form.



(b). Given that $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$. Show that $\nabla^2\left(\ln|\underline{r}|\right) = \dfrac{1}{|\underline{r}|^2}$

(c). Find the unit normal to the surface given by the function $2x^2 + 4yz - 5z^2 = -10$ at the point P with coordinates $(3, -1, 2)$

Q2(a). A particle P moves along a curve whose parametric equations are $x = k(1-\cos t)$, $y = k(t+\sin t)$ where t is the time. Show that (i). The magnitude of the velocity at any time t is given by $2k\cos\left(t/2\right)$

(ii). The magnitude of its acceleration at any time t is given by k

(b). An engine boat cruising at a speed of v_0 (constant acceleration) shuts off its engine at $t = 0$ and begins to decelerate with a magnitude of bv where v is the speed of the boat at any time $t > 0$ and b is a positive constant. Determine

(i) the velocity of the boat at any time t

(ii). How far the boat will travel before it eventually comes to a stop.

(c). Define the following in the cartesian coordinate system:

(i). Gradient of a scalar function $\phi(x, y, z)$

(ii). Divergence of a vector function $\underline{v}(x, y, z)$

(iii). Curl of a vector function $\underline{F}(x, y, z)$

Q3(a). A particle moves so that its position vector at any time t is given by

$\underline{r} = e^{-t}\cos t\underline{i} + e^{-t}\sin t\underline{j}$. Show that at any time t

(i). The velocity vector of the particle is inclined at its position vector at a constant angle of $\frac{3}{4}\pi$ radians.

(ii). The acceleration vector of the particle is at right angle to the position vector of the particle.

(b). A rectangular coordinate system transforms into a spherical coordinate system by the transformations $x = \rho\sin\theta\cos\phi$, $y = \rho\sin\theta\sin\phi$ and $z = \rho\cos\theta$. By using $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$, determine the following:

(i) (α) h_ρ (β) h_θ (γ) h_ϕ

(ii) (α) $\hat{e}_\rho$ (β) $\hat{e}_\theta$ (γ) $\hat{e}_\phi$

(c). Use the information in **(b)** above to define the operator del (∇) in the spherical coordinate system (ρ, θ, ϕ)

Q4(a). Given that, the transformation from the rectangular coordinate system (x, y, z) to the spherical coordinates system (r, θ, ϕ) is defined as follows: $z = \rho\cos\phi$ $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, and $\rho \geq 0$, $0 \leq \theta \leq 2\pi$ and $0 < \phi < \pi$. Show that

(i). $\rho = f(x, y, z)$

(ii). $\theta = g(x, y)$

(iii). $\phi = h(\rho, z)$

(b). A particle of mass $4kg$ rests in limiting equilibrium on a rough plane incline $30^{\bullet}$ to the horizontal. Find the coefficient of friction between the particle and the plane and leave your answer to three significant figures .

(c). The line of action of a force $\underline{F} = 5\underline{i} - 6\underline{j} + 2\underline{k}$ passes through the point A with position vector $\underline{r}_A = 2\underline{i} - \underline{j} + \underline{k}$. Find the magnitude of the moment of the force about the line l where $l : \underline{r} = 4\underline{i} + \underline{j} - 3\underline{k} + \lambda(-\underline{i} + 2\underline{j} - 2\underline{k})$. Distances are in metres and forces in Newtons.

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