UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

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COURSE NO: MA 173
COURSE NAME: VECTOR ANALYSIS
CLASS: MA I **TIME:** 3 HOURS

Name**:** _____Index Number: _____

ANSWER THREE QUESTIONS

Q1 (a). Two vectors $\underline{a}$ and $\underline{b}$ are given by $\underline{a} = m\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{b} = -2\underline{i} + \frac{1}{2}\underline{j} - 4\underline{k}$. If the angle between them is $\cos^{-1}\left(\frac{1}{3}\right)$, find the value of m if m is an integer.

(b). The line l_1 passes through the point with position vector $\underline{i} + 9\underline{k}$ and is parallel to $-\underline{i} - 4\underline{j} + 3\underline{k}$. The line l_2 passes through the point with position vector $\underline{i} + 2\underline{j} + 3\underline{k}$ and is parallel to the vector $\underline{i} + \underline{j} - \underline{k}$. Write down the vector equations of the lines l_1 and l_2.

Q2. The position vectors of the points A, B, C and D relative to an origin are respectively $\underline{a} = \underline{i} - 3\underline{j} + 2\underline{k}$, $\underline{b} = 2\underline{i} + 4\underline{j} - \underline{k}$, $\underline{c} = 9\underline{i} + 23\underline{j} - 4\underline{k}$ and $\underline{d} = 4\underline{i} + 3\underline{j} + 2\underline{k}$. L is the point which divides AB internally in the ratio 1: 2 and M is the midpoint of CD. Find

(a). The vector equations of AB and CD, and hence the position vector of their point of intersection F.

(b). The position vectors of L and M.

Q3(a). State the following laws associated with vector addition and illustrate them with approprite diagrams:

(i) The parallelogram law (ii) The triangle law

(b). A vector $\underline{a}$ has direction cosines (λ, μ, ρ) and a vector $\underline{b}$ has direction cosines (λ', μ', ρ'). Show that if θ is the angle between $\underline{a}$ and $\underline{b}$ then $\cos\theta = \lambda\lambda' + \mu\mu' + \rho\rho'$

Q4(a) Given that $\underline{n}$ is the normal vector to a plane and $\underline{d}$ is the direction vector of a line, show that the angle θ between the line and the plane is given by:

$$\theta = \sin^{-1}\left[\frac{\underline{n} \bullet \underline{d}}{|\underline{n}\|\underline{d}|}\right]$$

(b) Show that the acute angle between the line l and the plane P whose equations are:

$l : \underline{r}_1 = \underline{i} + 7\underline{j} + 5\underline{k} + \lambda(3\underline{i} + 4\underline{j} + 12\underline{k})$ and

$p : \underline{r}_2 = \underline{i} - \underline{j} + \mu(3\underline{i} + 2\underline{j} + 2\underline{k}) + \rho(\underline{j} + 2\underline{k})$ respectively is $\sin^{-1}\left(\frac{18}{91}\right)$.

***Examiner: Dr Mrs C.C. Nyarko** / MR P.K.Nyarko*