UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/ DEC 2018

COURSE NO: GM/GL/MN/MR/PE/ES 255

COURSE NAME: MATHEMATICAL METHODS

CLASS: GM/GL/MN/MR/PE/ES II **TIME:** 3 HOURS

Name**:** ____ Index Number: _______________

Answer ALL questions in Section A and TWO (2) questions in Section B. Carefully read each question in Section A and circle the LETTER of the correct answer on the Question Paper provided. For the Section B, All working must be clearly done in the Answer Booklet

SECTION A

1. A solution of the initial value problem $y' + 8y = 1 + e^{-6t}, \quad y(0) = 0$ is
   a. $y(t) = \frac{1}{8} + \frac{1}{2}e^{6t} - \frac{5}{8}e^{8t}$
   b. $y(t) = 4 - e^{-2t} + 3e^{-8t}$
   c. $y(t) = 4 - e^{2t} + 3e^{8t}$
   d. $y(t) = \frac{1}{8} + \frac{1}{2}e^{-6t} - \frac{5}{8}e^{-8t}$

2. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^3 + \sin(y) + 1 = 0$ is
   a. 2
   b. 3
   c. 5
   d. 6

3. The value of the integral $\int_0^1 \left(1 - x^{\frac{1}{n}}\right)^m dx$ is ………………………….
   a. $\beta(n,\ m+1)$
   b. $n\beta(n,\ m+1)$
   c. $\beta(m,\ n)$
   d. none of the above

4. The beta function can be expressed through gamma function as
   a. $B(x,y) = \frac{G(x)G(y)}{G(x+y)}$
   b. $B(x,y) = -\frac{G(x)G(y)}{G(x+y)}$
   c. $B(x,y) = \frac{G(x+y)}{G(x)G(y)}$
   d. $B(x,y) = -\frac{G(x+y)}{G(x)G(y)}$

5. A solution to a D.E. on an interval $\alpha < t < \beta$ is any function $y(t)$ which satisfies the D.E. on the interval
   a. $\alpha < t < \beta$
   b. $\alpha \le t < \beta$
   c. $\alpha < t \le \beta$
   d. $\alpha \le t \le \beta$

6. Initial conditions are of the form
   a. $y(t) = y_0 \text{ and } y^n(t) = y^{/}$
   b. $y(t_0) = y_0 \text{ and / or } y^n(t) = y_0^{/}$
   c. $y(t_0) = y_0 \text{ and / or } y^k(t_0) = y_k$
   d. A $y(t_0) = y_0 \text{ and } y^k(t) = y_k$

7. The most general form of a 1st order D.E. can be written as
   a.

b. $y = cy(t)$

c. $\dfrac{dy}{dt} = f(x,t)$

d. $y(t) = cy_1(t)$

e. $\dfrac{dy}{dt} = f(y,t)$

8. $M(x,y) + N(x,y)y' = 0$ is exact if and only if

a. $M_y \neq N_x$

b. $M_y = N_x$

c. $M_x = N_y$

d. $M_x \neq N_y$

9. The general constant coefficient, homogeneous, linear, second order D.E. is of the form

a. $p(t)y'' + q(t)y' + r(t)y = 0$

b. $p(t)y'' + q(t)y' + r(t)y = g(t)$

c. $ay'' + by' + cy = 0$

d. $ay'' + by' + cy = d$

10. The discriminant of real and distinct roots is of the form

a. $b^2 > 4ac$

b. $b^2 < 4ac$

c. $b^2 = 4ac$

d. $b^2 < -4ac$

11. A separable differential equation can be written in the form

a. $M(y)\dfrac{dy}{dt} = N(y)$

b. $N(y)\dfrac{dy}{dx} = M(x)$

c. $N(y)dx = M(x)dy$

d. $\dfrac{dy}{dx} = \dfrac{N(y)}{M(x)}$

12. The Laplace transform of $\cos at$ is given as

a. $\dfrac{a}{s^2 + a^2}$

b. $\dfrac{1}{s^2 + a^2}$

c. $\dfrac{a}{s - a^2}$

d. $\dfrac{s}{s^2 + a^2}$

13. The solution to $\dfrac{\Gamma\left(\frac{5}{2}\right)}{\Gamma\left(-\frac{1}{2}\right)}$ is

a. 30

b. $-\dfrac{3}{8}$

c. $\dfrac{4}{3}$

d. -30

14. The solution to $\int_0^1 x^4(1-x)^3\,dx$ is

a. $\dfrac{1}{280}$

b. $\dfrac{1}{120}$

c. $\dfrac{64\sqrt{2}}{15}$

d. $\dfrac{\sqrt{4}}{12}$

15. The inverse Laplace transform of $F(s) = \dfrac{6}{s} - \dfrac{1}{s-8} + \dfrac{4}{s-3}$ is

a. $6 - e^{3t} + 4e^{-8t}$

b. $6 - e^{8t} + 4e^{3t}$

c. $6 - e^{-3t} + 4e^{-8t}$

d. $6 + e^{-3t} - 4e^{-8t}$

16. Determine the order and degree of the differential equation $2x\frac{d^4y}{dx^4}+5x^2\left(\frac{dy}{dx}\right)^3-xy=0$.

a. Fourth order, first degree
b. First order, third degree
c. Third order, first degree
d. First order, fourth degree

17. Find the general solution of $\frac{dy}{dx}=x\cos x$.

a. $y=x\sin x+c$
b. $y=x\sin x-\cos x+c$
c. $y=x\sin x+\cos x+c$
d. $y=-x\sin x+c$

18. Solve the IVP $\frac{dy}{dx}=xy \qquad y(0)=1$.

a. $y=In\left(\frac{x^2}{2}\right)+c$
b. $y=e^{x^2/2+c}$
c. $y=e^{x^2/2}+e^c$
d. $y=In\left(\frac{2}{x^2}\right)+c$

19. Find the Laplace transform of $2t^2+3t+4$.

a. $\frac{4+3+4s^2}{s^2}$
b. $\frac{4+3+4s^2}{s^3}$
c. $\frac{4+3s+4s^3}{s^2}$
d. $\frac{4+3s+4s^2}{s^3}$

20. Which of the following equations is an exact differential equation?

a. $(x^2+1)dx-xydy=0$
b. $xdy+(3x-3y)dx=0$
c. $2xydx+(2+x^2)dy=0$
d. $x^2ydy-ydx=0$

21. The value of $\int_0^{\frac{\pi}{2}}\sin^9 xdx$ is

a. $\frac{120}{315}$
b. $\frac{128}{315}$
c. $\frac{125}{315}$
d. $\frac{120}{128}$

22. Which of the following equations is a variable separable differential equation?

a. $(x+x^2y)dy=(2x+xy^2)dx$
b. $2ydx=(x^2+1)dy$
c. $(x+y)dx-2ydy=0$
d. $y^2dx+(2x-3y)dx=0$

23. Find the Laplace transform of $2e^{-3t}-4e^{5t}$.

a. $\frac{-2(s+11)}{(s+3)(s-5)}$
b. $\frac{2(s-11)}{(s+3)(s-5)}$

c. $\dfrac{-2}{(s+3)(s-5)}$

d. $\dfrac{2}{(s+3)(s-5)}$

24. If $I_n = \int_0^{\frac{\pi}{2}} \sin^n x dx = \dfrac{n-1}{n} I_{n-2}$ then the value of I_2 is ………………………

a. $\dfrac{\pi}{2}$

b. $\dfrac{\pi}{4}$

c. $-\dfrac{\pi}{2}$

d. $-\dfrac{\pi}{4}$

25. The differential equation $x\dfrac{dy}{dx} = x - y$ is

I. **Separable** ***II.*** **Linear** ***III.*** **Homogeneous** ***IV.*** **Exact**

a. II, III, IV

b. I, II, III

c. I, II

d. II, IV

26. A differential equation is considered to be ordinary if it has

a. One dependent variable

b. More than one dependent variable

c. one independent variable

d. two independent variables

27. If $\phi(x,y)$ is a potential function for $M(x,y) + N(x,y)y' = 0$, then

a. $\phi_x = N(x,y)$ and $\phi_y = M(x,y)$

b. $\phi(x,y)$ is a solution of the DE

c. $M_x = N_y$

d. The general solution of the DE is $\phi(x,y) = C$

28. Solve $\dfrac{d^2y}{dx^2} + 8\dfrac{dy}{dx} + 16y = 0$

a. $y = c_1 e^{-2x} + c_2 e^{-4x}$

b. $y = c_1 e^{2x} + c_2 e^{4x}$

c. $y = (c_1 + c_2 x)e^{-4x}$

d. $y = (c_1 + c_2)e^{-4x}$

29. An integrating factor of the D. E. $(y/x - x) + y' = 0$ is

a. x

b. y

c. $In\, x$

d. x

30. What kind of differential equation is $t\dfrac{dy}{dt} = 3(y+1)^2 + 2$

a. Linear

b. Nonlinear

c. Homogeneous

d. Exact

SECTION B

Instruction: Answer **ONLY TWO** questions from this section

Question 1.

a) i) The variation of resistance, R ohms of an aluminium conductor with temperature $\theta°c$ is given by $\frac{dR}{d\theta} = \alpha R$, where α is the temperature coefficient of resistance of aluminium. If $R = R_0$ when $\theta = 0°c$, solve the equation for R.

ii) If $\alpha = 38\times10^{-4}$, determine the resistance of an aluminium conductor at $50°c$, correct to 3 s.f. when its resistance at $0°c$ is 24.0Ω

b) If $f(t) = \begin{cases} 2, & 0 < t < 3 \\ t, & t > 3 \end{cases}$, find $L\{f(t)\}$.

Question 2.

a) The current in an electric circuit is given by the equation

$$Ri + L\frac{di}{dt} = 0$$

where L and R are constants. Show that $i = Ie^{-\frac{Rt}{L}}$, given that $i = I$ when $t = 0$.

b) Find the general solution to the differential equation $2y'' + 18y = 6\tan(3t)$

Question 3.

a) Solve the differential equation $\frac{dy}{dx} = e^{-2y-x}$ given that $y = 0$ when x=0.

b) When solving the D.E. $\frac{d^2\theta}{dt^2} - 6\frac{d\theta}{dt} - 10\theta = 20 - e^{2t}$ by Laplace transforms for given boundary conditions, the following expression for $L\{\theta\}$ results:

$$L\{\theta\} = \frac{4s^3 - \frac{39}{2}s^2 + 42s - 40}{s(s-2)(s^2 - 6s + 10)}$$

Show that the expression can be resolved into partial fractions to give:

$$L\{\theta\} = \frac{2}{s} - \frac{1}{2(s-2)} + \frac{5s-3}{2(s^2 - 6s + 10)}$$

Question 4:

(a). Using the generalization $\Gamma(n+1)=n\Gamma(n)$ together with the fact that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$, evaluate the following:

(i) $\Gamma\left(-\frac{1}{2}\right)$ (ii) $\Gamma\left(-\frac{5}{2}\right)$

(b). evaluate the following integral using your knowledge in Gamma and Beta functions:

(i). $\int_0^1 x^4\sqrt{1-x^2}\,dx$ (ii) $\int_0^{\frac{\pi}{2}} \sin^5\theta\cos^4\theta d\theta$

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