# UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

## SECOND SEMESTER EXAMINATIONS, MAY 2018

**COURSE NO**: MN/MR/GL/PE/ES 250
**COURSE NAME**: **MATHEMATICAL ANALYSIS**
**CLASS**: **MN/MR/GL/PE/ES II** TIME: 3 HOURS

Name:  Index Number: _____

**ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**QUESTION 1**

(a) When do we say a sequence is monotonically increasing or decreasing? **[3 marks]**

(b) Prove that the sequence with the *nth* term $u_n = \left(1+\frac{1}{n}\right)^n$ is monotonic, increasing, and bounded, and thus a limit exists. **[5 marks]**

(c) Verify the validity of the entries in the following table

| Sequence | Monotonic Increasing | Limit Exists |
|---|---|---|
| $0.6, 0.66, 0.666\ldots, \frac{2}{3}\left(1-1/10^n\right)$ | **Yes** | **Yes** $\left(\frac{2}{3}\right)$ |
| $-1, +2, -3, +4, -5, \ldots, (-1)^n n$ | **No** | **No** |

**[6 marks]**

(d) State the conditions governing the following test with reference to test of convergence and divergence of an infinite series.

i. Logarithmic Test

ii. De- Morgan's and Bertrand's Test

iii. Cauchy *nth* root test

iv. Raabe's Test

v. Limit Comparison Test

vi. Alternating Series Test

**[12 marks]**

(e) Two students from UMaT were asked to write an $nth$ term for the sequence $1, 16, 81, 256, \ldots$ and also write the $5th$ term of the sequence. Student A gave the $nth$ term as $U_n = n^4$. Student B, who did not recognise this simple law of motion, write $U_n = 10n^3 - 35n^2 + 50n - 24$. Which student gave the correct $5th$ term? Justify your answer. **[4 marks]**

**QUESTION 2**

(a) State the difference between Taylor's Series and Taylor's Polynomial **[2 marks]**

(b) Find the Interval of convergence and the radius of convergence for the following power series

i. $\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$ ii. $\sum_{n=1}^{\infty} (-1)^n \frac{(x+4)^n}{n6^n}$

**[10 marks]**

**(c)** State Rolle's theorem and the Mean Value theorem **[3 marks]**

**QUESTION 3**

**(a)** State two theorems of Power Series **[2 marks]**

(b) Find the Maclaurin's Series for the function $f(x) = (1+x)^k$ ,express your answer in a sigma notation and hence determine the radius of convergence. **[6 marks]**

**(c)** Prove Taylor's Series and Maclaurin's Series using the concept in Power Series. **[5 marks]**

(d) Find the extrema of $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$ **[2 marks]**

**QUESTION 4**

(a) Prove that $\lim_{n\to\infty} U_n$ exists, it must be unique **[4 marks]**

(b) Find the fourth Taylors Polynomial for the function $f(x) = Inx \ at \ x = 1$ **[4 marks]**

**(c)** Using the Ratio Test determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{10^n}{e^n (n+1)!}$ **[5 marks]**

(d) Define the term Relative Extrema **[2 marks]**

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