Mathematical Analysis

MC/EL/CE 260 **Time: Three and half hours**

Instructions: Provide only the final answer in the booklet. You are supposed to submit both the question paper together with the answer booklet.

SECTION A

1. Write down the sum of the first n terms of the series and deduce it's sum to infinity

   $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

2. Find the sum of the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\ldots$

3. Find the sum of the series $\sum_{n=0}^{\infty}\frac{1}{n^2+3n+2}$

4. Find the sum of the series $\sum_{n=0}^{\infty}\frac{1}{n^2+4n+3}$

SECTION B

Determine whether the following series convergences or divergences

5. $\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

6. $\sum_{n=0}^{\infty}ne^{-n^2}$

7. $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$

8. $\sum_{n=1}^{\infty}\frac{n^2}{\sqrt{n^3+7}}$

9. $\sum_{n=1}^{\infty}\frac{\ln n}{n}$

10. $\sum_{n=2}^{\infty}\frac{1}{\sqrt[3]{n}-1}$

11. $\sum_{n=1}^{\infty}\left(\frac{\cos(n)}{n}\right)^2$

12. $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{2n^3+n}$

13. $\sum_{n=1}^{\infty}\frac{6n^2 2^n}{n^4+3}$

14. $\sum_{n=2}^{\infty}\frac{\cos(n\pi)}{\sqrt{n}}$

15. $\sum_{n=1}^{\infty}\frac{(-10)^n}{4^{2n+1}(n+1)}$

16. $\sum_{n=1}^{\infty}\frac{8^{n+1}n^2}{2^{2n}}$

SECTION C

State the following test for convergence

17. Ratio test
18. Alternating Series Test
19. Limit Comparison test
20. Integral Test

SECTION D

21. **State the Taylor polynomial**
22. **State the Fubini's Theorem**
23. Find the Power series representation for the series for $f(x)=\frac{1}{x^2}$ *about* $x=-1$
24. Determine the radius of convergence and interval of convergence for the power series

$$\sum_{n=1}^{\infty}\frac{2^n}{n}(4x-8)^n$$

25. Determine the radius of convergence and the interval of convergence for the following series $\sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}(x+3)^n$
26. Find the first three nonzero terms of the Maclaurin Series for $e^{\cos x}$

27. Find the first two nonzero terms in the Maclaurin series for $\frac{x}{\sqrt{1-x^2}}$

28. Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{n(x+2)^n}{5^{n-1}}$

29. Evaluate the integral $\int_0^2 \int_{x^2}^4 \frac{x^3}{\sqrt{x^4+y^2}} dydx$

30. Find $\int_0^{\infty} t^3 e^{-4t} dt$

31. State the alternative form of the beta function

32. Solve the integral $I = \int_0^1 x^4 \sqrt{1-x^2}\, dx$

33. Evaluate $I = \int_0^1 x^5 (1-x)^4 dx$

34. Solve the $\int_0^{\infty} x^6 e^{-4x^2} dx$

Section E

Evaluate the integral over the indicated region

35. $\iint_R 6xy^2 dA, \quad R = [2,4] \times [1,2]$

36. $\iint_R 2x - 4y^3 dA, \quad R = [-5,4] \times [0,3]$

37. $\iint_R x^2 y^2 + \cos(\pi x) + \sin(\pi y)\, dA, \quad R = [-2,-1] \times [0,1]$

38. $\iint_R \frac{1}{(2x+3y)^2} dA, \quad R = [0,1] \times [1,2]$

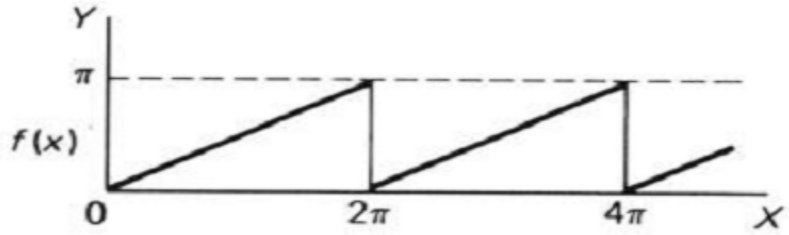
39. $\iint_R xe^{xy} dA, \quad R = [-1,2] \times [0,1]$

40. $\iint_D 4xy - y^3 dA$, D is the region bounded by $y = \sqrt{x}$ and $y = x^3$

41. Find the volume of the solid that lies below the surface given by $z = 16xy + 200$ and lies above the region in the xy-plane bounded by $y = x^2$ and $y = 8 - x^2$.

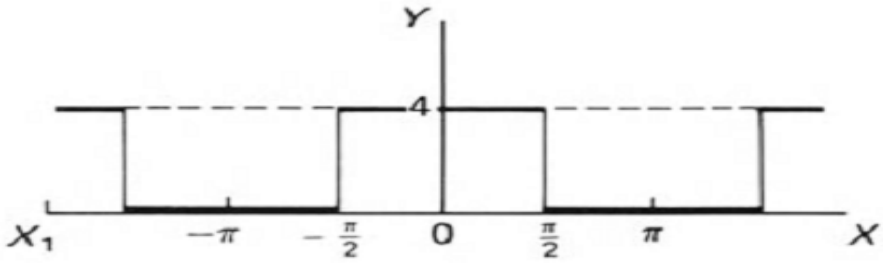
SECTION F

(A)



42. Find a_0 for figure A
43. Find a_n for figure A
44. Find b_n for figure A
45. Write the Fourier series representation for figure A

(B)



46. Find a_0 for figure B
47. Find a_n for figure B
48. Find b_n for figure B
49. Write the Fourier series representation for figure **B**
50. Find the gradient of the function $f(x,y)=x+y^2$
51. Find the gradient of the function $f(x,y,z)=\dfrac{1}{\sqrt{x^2+y^2+z^2}}$
52. Find the gradient of the function $f(x,y,z)=\sin(x)e^y\ln(z)$
53. Calculate the divergence of the vector field $F=e^x i+\ln(xy)\,j+e^{xyz}k$
54. Calculate the divergence of the vector field $G=\dfrac{4y}{x^2}i+\sin(y)\,j+3k$
55. Calculate the divergence of the vector field $G=x^2i+2zj-yk$
56. Find the volume enclosed by the cardioid with radius $r=1+\cos\theta$
57. Find the curl of the vector field $F=y^3i+xyj-zk$
58. Find the curl of the vector field $F=y^3i+xyj-zk$
59. Find the curl of the vector field $F=\dfrac{xi+yj+zk}{\sqrt{x^2+y^2+z^2}}$

60. Evaluate the double integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} x dy dx$