UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC. 2018

COURSE NO: EL/ MC/ CE/ RN 169
COURSE NAME: LINEAR ALGEBRA
CLASS: EL/ MC/ CE/ RN I

TIME: 3 HOURS

Name**:** _____ Index Number: _____

Write only the final answer on the question paper and do the rough work in the answer booklet.
The section B should be done in the answer booklet on a new page

Section A

1. If $u = 2\left(\cos -\frac{\pi}{6} + i\sin -\frac{\pi}{6}\right)$ then find $\bar{u}^{-1}$
2. Simply $\left(1+i\sqrt{3}\right)^6 + \left(1-i\sqrt{3}\right)^6$
3. Solve the equation $z^2 + 2iz + 1 = 0$

Use these to answer questions 4 to 5 leaving your answer in polar form

If $z_1 = 3\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$, $z_2 = 2\left(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18}\right)$ and $z_3 = \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

4. Find $\frac{z_1^2 z_3}{z_2}$
5. Find $z_1 z_2 z_3$
6. Find the third root of the equation $z^3 - 1 + i = 0$
7. Simplify $\frac{-1+3i}{(3-2i)(2+i)}$ leaving your answer in polar form.
8. Evaluate $\left(\sqrt{3}+i\right)^6$
9. Find the values of x and y which satisfies the equation $\frac{(1+i)x-2i}{3+i} + \frac{(2-3i)y+i}{3-i} = i$
10. Find the modulus and argument of $\frac{2-i}{3+i} + \frac{1+i}{1-i}$

11. If $z=\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)^5+\left(\frac{\sqrt{3}}{2}-\frac{i}{2}\right)^5$ then find $\mathrm{Im}(z)$

12. If $i=\sqrt{-1}$ then $4+5\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)^{334}+3\left(\frac{-1}{2}+i\frac{\sqrt{3}}{2}\right)^{365}$ is equal to

13. If $\arg(z)<0$, then $\arg(-z)-\arg(z)=$

14. If $|z|=1$ and $w=\frac{z-1}{z+1}$ (where $z\neq -1$) then $\mathrm{Re}(w)$ is

15. If $x+iy=\sqrt{\frac{a+ib}{c+id}}$, Find $\left(x^2+y^2\right)^2$

16. Express $\frac{1}{(1-\cos\theta)+2i\sin\theta}$ in standard form

17. Given that $A=\begin{pmatrix}3+2i & 4i\\ 4-2i & -3\end{pmatrix}$ and B is a conjugate transpose of A. Then Find AB

18. If $A=\begin{pmatrix}2 & 3 & 5\\ 1 & 7 & 4\\ 8 & 0 & 6\end{pmatrix}$, Find A^TI

19. Find the adj of $A=\begin{pmatrix}1 & 4 & 3\\ 6 & 2 & 5\\ 1 & 7 & 0\end{pmatrix}$

20. Find the inverse $A=\begin{pmatrix}2 & 1 & 4\\ 3 & 5 & 1\\ 2 & 0 & 6\end{pmatrix}$

Use matrix $A=\begin{pmatrix}1 & 2 & 3\\ 1 & 3 & 1\\ 2 & 2 & 1\end{pmatrix}$ to answer questions 21 ton 22

21. Find the eigenvalues of matrix A

22. Determine the correspondent eigenvectors of A

23. Find the value of k which satisfies the equation $\begin{vmatrix}k & 1 & 0\\ 1 & k & 1\\ 0 & 1 & k\end{vmatrix}=0$

24. Solve the equation $\begin{vmatrix}x & 2 & 3\\ 2 & x+3 & 6\\ 3 & 4 & x+6\end{vmatrix}=0$

25. A resistive network gives the following equations

$$2(i_3 - i_2) + 5(i_3 - i_1) = 24$$

$$(i_2 - i_3) + 2i_2 + (i_2 - i_1) = 0$$

$$5(i_1 - i_3) + 2(i_1 - i_2) + i_1 = 6$$

Simplify and use the determinants to find the value of i_2 to two significant figures.

26. Determine the value of k for which the following equations have solutions

$$4x - (k-2)y - 5 = 0$$

$$2x + y - 10 = 0$$

$$(k+1)x - 4y - 9 = 0$$

27. Solve the equation $\begin{vmatrix} x+1 & -5 & -6 \\ -1 & x & 2 \\ -3 & 2 & x+1 \end{vmatrix} = 0$

28. Three currents i_1, i_2, i_3 in a network are related by the following equations

$$2i_1 + 3i_2 + 8i_3 = 30$$

$$6i_1 - i_2 + 2i_3 = 4$$

$$3i_1 - 12i_2 + 8i_3 = 0$$

Solve for the values of i_1, i_2, i_3 using determinants.

29. Find the cofactors of $\begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix}$

Use the following matrix $B = \begin{pmatrix} 2 & 3 & 5 \\ 4 & 1 & 6 \\ 1 & 4 & 0 \end{pmatrix}$ to solve questions (30) to (32)

30. Find $|B|$

31. Find $adj\, B$

32. Find the B^{-1}

33. Given that $p = 2i + 4j + 3k$ and $q = i + 5j + -2k$,find $p \times q$

34. Find the scalar product of $a = i + 2j + -k$ and $b = 2i + 3j + k$

35. If the position vectors of p and q are $i + 3j + -7k$ and $5i + -2j + 4k$ respectively, find $\overline{PQ}$ and determine its direction cosines.

36. Find the value of the parallelogram bounded by $\underline{a} = 2i + 4j + 3k$, $\underline{b} = 3i + k$ and $\underline{c} = i + j - k$

37. Two vectors are perpendicular if and only if

38. What is a skew symmetric matrix?

39. Find the angle between the two vectors $\underline{a} = 2i + 4j + -k$ and $\underline{b} = 3i - 2j + k$

40. Given that $\underline{a} = a_1 i + a_2 j + a_3 k$ and $\underline{b} = b_1 i + b_2 j + b_3 k$, find $\underline{a} \times \underline{b}$

(40 marks)

SECTION B

Answer all

1. Currents of i_1, i_2, i_3 in a network are related by the following equations:

$$z_1 i_1 + z_3 i_3 = v$$
$$z_2 i_2 - z_3 i_3 = 0$$
$$i_1 - i_2 - i_3 = 0$$

Determine expression for i_1, i_2, i_3 in terms of z_1, z_2, z_3 and v. **(15 Marks)**

2. Find the eigenvalues and eigenvectors for $A = \begin{pmatrix} 2 & 0 & 1 \\ -1 & 4 & -1 \\ -1 & 2 & 0 \end{pmatrix}$ **(15 Marks)**

3. Find all the roots for $z = \left(-1 - \sqrt{3}i\right)^{\frac{2}{3}}$ **(10 Marks)**

Examiner: **Henry Otoo**