

UNIVERSITY OF MINES AND TECHNOLOGY, TARKW
SECOND SEMESTER EXAMINATIONS, MAY 2018

COURSE NO : GM 174

COURSE NAME: SOLID GEOMETRY

CLASS : GM I TIME: 3 HOURS

Answer Question One and any other Two Question.

Q1. (a) Find the vector equation of the line through the points A and B with position vectors

$\underline{a} = -11\underline{i} + 17\underline{j}$ and $\underline{b} = 4\underline{i} - 3\underline{j} - 5\underline{k}$ respectively. The vector $7\underline{i} - 4\underline{k}$ is perpendicular to the plane through the point D with position vector $\underline{d} = 3\underline{i} + 5\underline{j} + 12\underline{k}$. Find the sine of the angle between the line AB and the plane. Show that the equation of the plane is $7x - 4z + 27 = 0$

(b). Show that the acute angle between the line L and the plane P whose equations are:

$$l : \underline{r}_1 = \underline{i} + 7\underline{j} + 5\underline{k} + \lambda(3\underline{i} + 4\underline{j} + 12\underline{k}) \quad \text{and}$$

$$p : \underline{r}_2 = \underline{i} - \underline{j} + \mu(3\underline{i} + 2\underline{j} + 2\underline{k}) + \gamma(\underline{j} + 2\underline{k})$$

respectively is $\sin^{-1}\left(\frac{18}{91}\right)$.

(c). Find the angle between the line with equation $\underline{r} = (2\underline{i} + \underline{k}) + \lambda(3\underline{i} - 4\underline{j} + \underline{k})$ and the plane with equation $\underline{r}(5\underline{i} + \underline{j} - 6\underline{k}) = 2$

Q2. (a). Find the direction cosines for the vector $\underline{a} = \underline{i} - 2\underline{j} + 3\underline{k}$

(b) Find the equation of a sphere for which the circle $x^2 + y^2 + z^2 = 4$, and the plane $x + y + 4 = 0$ is a great circle

(c). Find the equation of the sphere passing through the origin, and points $(0, 2, -1)$, $(-1, 1, 0)$, and $((1, 2, -3)$.

Q3. (a). Prove by illustrating the cosine formula for a spherical triangle ΔABC with sides a, b , and c , and internal angles \hat{A}, \hat{B} and \hat{C} .

(b). using the cosine formula in 3(a) above, prove the sine formula for spherical triangles.

(c). Two planes A and B on the earth have respectively the latitude and the longitude $(40^\circ N, 18^\circ E)$ and $(0^\circ N, 58^\circ E)$. Find the angle departure from A to B of the great circle route

Q4. (a). Using the definition of the versine $A=1-\cos A$, show that the natural haversine

formula is given by
$$\text{hav}A = \frac{\text{hav } a - \text{hav}(b \sim c)}{\sin b \sin c}$$
.

(b). In a spherical triangle ABC , show that $\sin \frac{A}{2} = \sqrt{-\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$, where

$$s = \frac{1}{2}(a + b + c).$$

(c). In a spherical triangle PZX , given that $p = 87^{\circ}10'$, $z = 62^{\circ}37'$, and $x = 100^{\circ}10'$.

Find P , Z , and Z