UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, APRIL/MAY 2019

COURSE NO: MN/MR/GL/PE/ES 250
COURSE NAME: MATHEMATICAL ANALYSIS
CLASS: MN/MR/GL/PE/ES II **TIME**: 2 HOURS

Name**:** _____Index Number: _____

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION 1

(a) Define the following terms as applied in Mathematical Analysis

i. Relative Extrema
ii. Critical Number
ii. Concavity
iv. Point of inflection **[6 marks]**

(b) What is the relationship between the Remainder term, Taylor series and the Taylor polynomial? **[2 marks]**

(c) Test for the convergence of the following series

i. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{3n-1}$ ii. $\sum_{n=1}^{\infty} \frac{e^{\tan^{-1} n}}{n^2+1}$ **[12 marks]**

QUESTION 2

(a) Assuming the **Ethiopian Aircraft** crashed due to system failure. If two statisticians observed that the system was generating systematic values $1, 16, 81, 256, \ldots$ Unfortunately, statistician A gave the nth term as $U_n = n^4$. Statistician B, who did not recognise this simple law of motion, write his nth term as $U_n = 10n^3 - 35n^2 + 50n - 24$. Which statistician gave the correct $5th$ term? Justify your answer.

[4 marks]

(b) State the conditions governing the following test with reference to test of convergence and divergence of an infinite series.

i. Logarithmic Test
ii. Raabe's Test
iii. Gauss Test
iv. Cauchy *nth* root test
v. Quotient Test

[10 marks]

(c) Show that $\tan x$ is strictly monotonic with domain $\left[-\frac{1}{2}\pi, \frac{1}{2}\pi\right]$ **[6 marks]**

QUESTION 3

(a) Define Rolle's theorem and the Lagrange's Mean Value theorem **[4 marks]**

(b) Verify Rolles theorem for $f(x) = 2x^3 + x^2 - 4x - 2$ **[4 marks]**

(c) Find the interval and radius of convergence for the following power series

i. $\sum_{n=1}^{\infty} \frac{2^n (x-5)^n}{n^2}$ ii. $\sum_{n=1}^{\infty} n!(x-a)^n$ **[12 marks]**

QUESTION 4

(a) Find the fourth Maclaurin's Polynomial for the function $f(x) = \sin 2x$ **[4 marks]**

(b) Find Taylors series for $f(x) = 3x^5 - x^4 + 2x^3 + x^2 - 2 \quad at \ x = -1$ **[4 marks]**

(c) Define the following terms as known in Mathematical Analysis

i. Radius of convergence

ii. Bounded sequence

iii. Oscillatory sequence

[6 marks]

(d) Discuss the applicability of Rolles theorem to the function

$$f(x) = \log\left\{\frac{x^2 + ab}{(a+b)x}\right\} \text{ in } (a,b), 0 \notin [a,b]$$

[6 marks]

Examiners: B. Odoi/ *H. Otoo*