UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY, 2019

COURSE NO: CE/EL/MC/RN 164

COURSE NAME: CALCULUS

CLASS: CE/EL/MC/RN 1 **TIME:** 3 HOURS

Name**:** _____Index Number: _____

*Attempt all questions in Section A and any **THREE** from Section B. Each question in Section A attracts 1 mark and each question in Section B attracts 20 marks. Answer Section A on the question paper and Section B in the Answer Booklet*

SECTION A

1. Find the derivative with respect to x of the function $h(x)=f(x)+g(x)$ if $f(x)=4x^3$ and $g(x)=14x^2$.

   a. $12x^3+10x$ b. $12x^2+10x$ c. $12x^3+28x$ d. $28x^3+10x$

2. Given that $y=4u^2+1$ and $u=3x^2+1$. Find $\frac{dy}{dx}$

   a. $48x(4x^2+1)$ b. $48x^2(3x^2+1)$ c. $48x(3x^2+1)$ d. $48x^2(4x^2+1)$

3. If functions f and g are such that $f(x)=g(x)+k$, where k is a constant then

   a. $f'(x)=g'(x)+k$

   b. $f'(x)=g'(x)$

   c. $f'(x)=g(x)+k$

   d. $f'(x)=\left[g'(x)\right]^2+k$

4. $\lim_{x\to 0}\frac{e^x-1}{x}$ is equal to

   a. 1 b. 0 c. $\frac{1}{2}$ d. 2

5. For what values of x does the graph of $f(x)=x^3-x^3+3$ have a horizontal tangent.

   a. 0 b. 0 and 3 c. 2 d. 0 and 2

6. Find $k'(s)$ if $k(x)=\frac{\ln s}{s^2}$

   a. $\frac{1}{25^2}$ b. $\frac{-2}{s^4}$ c. $\frac{1}{s^3}$ d. $\frac{2\ln s}{s^3}$

7. Evaluate $\int(5x-4)^6\,dx$

   a. $\frac{(4x-5)^7}{35}+c$ b. $\frac{(4x+5)^7}{3}+c$

c. $\frac{(5x+4)^7}{35}+c$ d. $\frac{(5x-4)^7}{35}+c$

8. $\int e^{4x}dx =$

a. $\frac{4e^{4x}}{4}+c$ b. $\frac{e^{4x+1}}{4}+c$ c. $\frac{e^{4x}}{4}+c$ d. $\frac{e^{4x-1}}{4}+c$

9. $\int \tan x dx =$

a. $-\ln\sin x+c$ b. $-\ln\cos x+c$ c. $-\ln\sec^2 x$ d. $-\ln\sec x+c$

10. ${}^nC_r$ is the same as

a. $\frac{(n-r)!}{n!}$ b. $\frac{n!}{r!(n-r)}$ c. $\frac{r!}{(n-r)!}$ d. $\frac{n!}{(r-n)!}$

11. $\lim_{x\to 0}\frac{(\cos x-1)}{x} =$

a. 0 b. 1 c. -1 d. 2

12. $\int ax^2dx = ...$

a. $\frac{(n+1)x}{a(n+1)}+c$ b. $\frac{x^{n+1}}{a(n+1)}+c$ c. $\frac{a\,x^{n+1}}{n+1}+c$ d. $\frac{x^{n+1}}{n+1}+c$

13. Differentiate with respect to x $\frac{\sin 2x}{2x+5}$

a. $\frac{(2x+5)2\cos 2x-2\sin 2x}{(2x+5)^2}$ b. $\frac{\sin 2x(2x+5)-\cos 2x}{(2x+5)^2}$

c. $\frac{(2x+5)2\cos 2x+2\sin 2x}{(2x+5)^2}$ d. $\frac{(2x+5)^2+\cos 2x}{(2x+5)^2}$

14. if $x^2+2xy+3y^2=4$, find $\frac{dy}{dx}$

a. $\frac{-(x^2+y^2)}{y^2+2xy}$ b. $\frac{-(x+y)}{(x+3y)}$ c. $\frac{(x+y)}{(x+3y)}$ d. $\frac{x^2+y^2}{y^2+2xy}$

15. if $y=\cos 2t$, $x=\sin t$ find $\frac{dy}{dx}$

a. $-4\sin t$ b. $-4\cos t$ c. $4\sin t$ d. $4\cos t$

16 . $\int \cos^2 x dx =$

a. $\frac{1}{4}\sin 2x - \frac{1}{2}x + c$ b. $\frac{1}{2}x - \frac{1}{4}\sin 2x + c$

c. $\frac{1}{4}\cos 2x - \frac{1}{2}x + c$ d. $\frac{1}{2}x + \frac{1}{4}\cos 2x + c$

17. Evaluate $\int \sin^3 x dx$

a. $\frac{1}{3}\cos^3 - \cos x + c$ b. $\frac{1}{3}\cos^2 - \cos x + c$

c. $\frac{1}{3}\cos^4 - \sin x + c$ d. $\frac{1}{3}\cos^4 - \sin x + c$

18. Find the derivative with respect to x of $e^{2x}\ln 5x$

a. $e^{5x}\left(\frac{1}{x} + 2\ln 5x\right)$ b. $e^{x}\left(\frac{1}{x} + 2\ln 5x\right)$

c. $e^{2x}\left(\frac{1}{x} + 2\ln 5x\right)$ d. $e^{5x}\left(\frac{1}{x} - 2\ln 5x\right)$

19. if $y = x^3.\sin x$. Find $\frac{dy}{dx}$

a. $2x^2\left(x\cos 3x + \sin 3x\right)$ b. $3x^2\left(x\cos 3x + \sin 3x\right)$

c. $2x^2\left(x\cos 3x - \sin 3x\right)$ d. $2x^2\cos 3x - \sin 4x$

20. If $y = \left(4x-5\right)^6$ then find $\frac{dy}{dx}$

a. $32\left(4x-5\right)^5$ b. $28\left(4x-5\right)^6$ c. $24\left(4x-5\right)^6$ d. $32\left(4x-5\right)^7$

21. If $f\left(x\right) = \cos^{-1} x$ then $\frac{dy}{dx} =$ ……

a. $\frac{-1}{\sqrt{1-x^2}}$ b. $\frac{1}{1+x^2}$ c. $\frac{1+x^2}{2}$ d. $\frac{1}{\left(1+x^2\right)^2}$

22. If $z=\sin(3x+2y)$, then $\frac{\partial z}{\partial x}=$ .....

a. $3\cos(3x+2y)$ b. $7\sin(3x+2y)$ c. $2\cos(3x+2y)$ d. $3\cos(3x^2+1)$

23. $\lim_{x\to 0}\left(\frac{x-\sin x}{x^2}\right)=$ .....

a. 1 b. $\frac{1}{2}$ c. $-\frac{1}{2}$ d. 0

24. $\int(4x^3+5x^2-2x+7)dx$

a. $x^4+\frac{5x^3}{3}-x^2+7x+c$ b. $12x^2+6x-2+c$

c. $\frac{4}{3}x^3+10x-2$ d. $x^4+\frac{5x^3}{3}-2x+7+c$

25. $\int e^{3x}dx=$ .....

a. $\frac{1}{3x}e^x+c$ b. $\frac{1}{3}e^x+c$ c. $\frac{1}{3}e^{3x}+c$ d. $\frac{1}{3}e^{2x}+c$

26. Find the derivative of the function $y=2^x$ using logarithmic differentiation.

a. $\frac{\ln 2}{2x}$ b. $2^x\ln c$ c. $2^x\ln(x)$ d. x^{2x-1}

27. Find $\frac{dy}{dx}$ for the function $x^2+y^2=4$

a. $\frac{dy'}{dx}=\frac{-y}{x}$ b. $\frac{dy'}{dx}=-2x+y$ c. $\frac{dy'}{dx}=\frac{x}{y}$ d. $\frac{dy'}{dx}=\frac{-x}{y}$

28. Find y' implicitly for the function $y^2=x$

a. $\frac{1}{2y}$ b. $-\frac{1}{2y}$ c. $2y$ d. $\sqrt{x}$

29. Which of the following is indefinite integral of x^2+7.

a. $\int(x^2+7)dx=2x+c$ b. $\int(x^2+7)dx=x^3+7x$

c. $\int\left(x^2+7\right)dx=\frac{1}{2}x^3+7x$ d.

$$\int\left(x^2+7\right)dx=\frac{1}{3}x^3+7x+c$$

30. Which of the following correctly evaluate the definite integral $\int_1^3\left(x^2+3x+2\right)dx$

a. $\frac{93}{7}$ b. $\frac{110}{3}$ c. $\frac{74}{3}$ d. 4

31. Given the differentiable function $f(x)$ and $g(x)$, then $\frac{d}{dx}\left[f(x)g(x)\right]=$

…….

a. $g'(x)f'(x)+f'(x)g'(x)$ b. $\left[g(x)\right)^2-\left[f(x)\right]^2$

c. $g'f(x)+g(x)f'(x)$ d. $\frac{f(x)}{g(x)}+f'(x)$

32. If $y=\ln\left(5x^2+6x-8x^4\right)$ then $\frac{dy}{dx}=$……..

a. $5x^2+6x+10$ b. $-32x^3+10x+6$

c. $\frac{-8x^4+6x+5x^2}{10x+6-32x^3}$ d. $\frac{-32x^3+10x+6}{-8x^4+6x+5x^2}$

33. In the partial fraction decomposition $\frac{x+3}{(x+5)(x-8)}=\frac{A}{x+5}+\frac{B}{x-8}$. Find B

a. $\frac{11}{13}$ b. $\frac{2}{13}$ c. -1 d. 1

34. Which of the following theorem or rule is used when verifying Rolle's theorem

a. Product Rule b. Quotient rule c. Extreme value theorem d. kinetic theorem

35. The integral of $\sin x$ is

a. $\cos x$ b. $-\sin x$ c. $-\cos x$ d. $\sin^2 x$

36. $f(x,y)=3xy^2-2y+5x^2y^2$ find $\frac{\partial F}{\partial x}$

a. $10xy^2+2$ b. $3y^2+10xy^2$ c. $10x^3+5y^2$ d. $10y^2+5$

37. A partial differential equation requires

a. exactly one independent variable

b. two or more independent variables

c. more than one dependent variable

d. equal number of dependent and independent variable

38. Find the derivative with respect to x of $\sec^{-1} x$

a. $\frac{1}{\sqrt{x^2-1}}$ b. $\frac{1}{x\sqrt{x^2-1}}$ c. $\frac{1}{1+x^2}$ d. $\frac{1}{\sqrt{x^2+1}}$

39. $f(x)=7x^{-4}$ find $f'(2)$

a. -14 b. -3.5 c. -0.87 d. -1.75

40. $g(x)=\frac{x^2+5}{x^2+6x}$

a. $g'(x)=\frac{2x^3-4x^2-30x}{x^2(x+6)}$ b. $g'(x)=\frac{6x^2-10x-30}{x^2(x+6)^2}$

c. $g'(x)=\frac{4x^3+18x^2+10x+30x}{x^2(x+6)^2}$ d. $g'(x)=\frac{x^4+6x^3+5x+30x}{x^2(x+6)^2}$

SECTION B

Question 1

a. Use the reduction formula to evaluate $\int x^5 \cos x dx$

b. Find $\frac{dy}{dx}$ given that;

i. $x^3 + y^3 + 3xy^2 = 0$

ii. $y = x\sin^{-1} x$

iii. $y = \tan^{-1}(2x-1)$

c. Expand the following in ascending powers of x using the Maclaurin's Series

i. $f(x) = \sin^2 x$ $f(x) = \tan x$

d. If $z = \frac{\sin(3x+2y)}{xy}$,find $\frac{\partial z}{\partial x}$

e. Use L'hopitals rule to find the $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

Question 2

a. Given that $f(x) = x^7 \cos x$ use Leibnitz theorem to find;

i. $\frac{d^2 f}{dx^2}$

ii. $\frac{dx^4 f}{dx^2}$

b. Given that $z = \frac{x^3 - y^3}{xy}$, show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$

c. Find $\frac{dy}{dx}$ given that

i. $x^3 + y^3 + 3xy^2 = 0$

ii. $y = x\sin^4 x$

iii. $y = \tan^{-1}(2x-1)$

d. If $y = \sin^{-1} x$ show that $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

Question 3

a. Find the stationary points on the following curve and determine the type;

i. $f(x) = \frac{1}{3}x^3 - 2x^2 + 3x$

ii. $y = \frac{x^3}{3} - \frac{x^2}{2} - 2x + 15$

b. Use the reduction formula to determine

i. $\int \sin^6 2x dx$

ii. $\int x^4 e^{2x} dx$

c. Evaluate the following

i. $\int \sin^3 x \cos^2 x dx$

ii. $\int \sin^2 x \cos^4 x dx$

d. Given that $z = 3x^2 - y^2$ where $x = 2s + 7t$ and $y = 5st$. Find $\frac{\partial z}{\partial t}$ and express it in terms of s and t.

Question 4

a. The parametric equation of a function are given as $y = \cos 2t$, $x = \sin t$. Find

i. $\frac{dy}{dx}$

ii. $\frac{d^2y}{dx}$

b. If $z = \ln\left(e^x + e^y\right)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

c. Evaluate $\int e^{ax} \sin bx dx$

d. Verify the mean value theorem for $f(x) = 2\sqrt{x}$ on $[1,4]$