UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATIONS, NOV/DEC 2018

COURSE NO: CE 275
COURSE NAME: DISCRETE MATHEMATICS
CLASS: CE II **TIME:** 2 HOURS

*Answer **TWO** Questions **ONLY** in this section*

1. a) Suppose that the universal set is U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Express each of these sets with bit strings.

(i) {3, 4, 5} (ii) {1, 3, 6, 10} (iii) {2, 3, 4, 7, 8, 9}

b) Using the same universal set, find the set specified by each of these bit strings.

(i) 11 1100 1111 (ii) 01 0111 1000 (iii) 10 0000 0001

c) Show how bitwise operations on bit strings can be used to find these combinations of $A=\{a,b,c,d,e\}$, $B=\{b,c,d,g,p,t,v\}$, $C=\{c,e,i,o,u,x,y,z\}$, $D=\{d,e,h,i,n,o,t,u,x,y\}$

i) $(A\cup B)$
ii) $A\cap B$
iii) $(A\cup D)\cap(B\cup C)$
iv) $A\cup B\cup C\cup D$

d) For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive

i) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$
ii) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$
iii) $\{(2,4),(4,2)\}$
v) $\{(1,2),(2,3),(3,4)\}$
vi) $\{(1,1),(2,2),(3,3),(4,4)\}$
vii) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

2 a) Without using the membership table approach, prove the identity

$$(A\cup B)\cap C=(A\cap C)\cup(B\cap C)$$

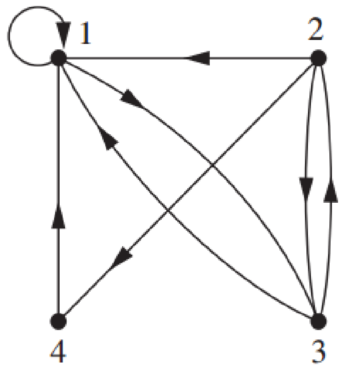
b) Prove the second De Morgan law in by showing that if *A* and *B* are sets, then

$$\overline{A\cup B}=\bar{A}\cap\bar{B}$$

i) by showing each side is a subset of the other side.
ii) by using a membership table.

c) Prove by induction that $1^2+2^2+3^2+\cdots+n^2=\frac{1}{6}n(n+1)(2n+1)$ for $n\geq 1$.

3. a) Find the transitive closure of the digraph below.



b) Prove by mathematical induction that $n^3 - n$ is divisible by 3 for all $n > 1$.

c) For $i = 1, 2, \cdots$, let $A_i = \{i, i+1, i+2, \cdots\}$. Find

i) $\bigcup_{i=1}^{n} A_i$ ii) $\bigcap_{i=1}^{n} A_i$

Examiner: E. Danso-Addo