



UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

SECOND SEMESTER EXAMINATIONS, MAY 2018

COURSE NO: MC/EL/CE/RN 166

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COURSE NAME: CALCULUS

CLASS: MC/EL/CE/RN I

TIME: 3 HRS

Name: _____ Index Number: _____

Answer **ALL** Questions in **Section A** and **Two (2)** Questions in **Section B**. Carefully read each question in **Section A** and write the letter of the correct answer in the **Answer Booklet** provided.

For the **Section B**, **All** working must be clearly done in the **Answer Booklet**.

1. What is the $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x}$?

- a. -1 b. 0 c. 1 d. 2

2. If $f(x) = x$, then $f'(5) =$ _____

- a. 0 b. $\frac{1}{5}$ c. 1 d. 5

3. $\int (x^2 + 1)^2 dx =$ _____

- a. $\frac{(x^2 + 1)^3}{3} + c$ b. $\frac{x^5}{5} + \frac{2x^3}{3} + x + c$ c. $\frac{(x^2 + 1)^3}{6x} + c$ d. $\frac{(x^2 + 1)^3}{3} + c$

4. If $x^3 + 3xy + 2y^3 = 17$, then $\frac{dy}{dx}$ in terms of x and y is _____

- a. $-\frac{x^2 + y}{x + y^2}$ b. $-\frac{x^2 + y}{x + y^2}$ c. $-\frac{x^2 + y}{x + 2y^2}$ d. $-\frac{x^2 + y}{2y^2}$

5. $\frac{1}{2} \int e^{t/2} dt =$ _____

- a. $e^{-t} + c$ b. $e^{-t/2} + c$ c. $e^{t/2} + c$ d. $2e^{t/2} + c$

- a. $\frac{7}{\sqrt{5}}$ b. $\frac{14}{\sqrt{5}}$ c. $\frac{15}{\sqrt{21}}$ d. $\frac{30}{\sqrt{21}}$
13. $\lim_{h \rightarrow 0} \frac{\ln(4+h) - \ln(4)}{h}$ is _____
- a. 0 b. $\frac{1}{4}$ c. 1 d. e
14. $\frac{d}{dx} \left(\frac{1}{x^3} - \frac{1}{x} + x^2 \right) =$ _____
- a. -6 b. -4 c. 0 d. 2
15. $\int \sec x \tan x \, dx =$ _____
- a. $\sec x + c$ b. $\frac{\tan^2 x}{2} + c$ c. $\frac{\sec^2 x}{2} + c$ d. $\tan x + c$
16. Find the exact value of $\lim_{x \rightarrow 0} \frac{\sqrt{3+x} - \sqrt{3}}{x}$.
- a. 0 b. $\sqrt{3}$ c. $\frac{1}{2\sqrt{3}}$ d. The limit does not exist
17. Let $f(x) = \frac{5}{2}x^2 - e^x$. Find the value of x for which the second derivative $f''(x)$ equals zero.
- a. $\ln 5$ b. $5e$ c. e^5 d. 0
18. Which of the following statements is true
- a. If $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$ exists, then f is differentiable at a
- b. If f is continuous at a , then f is differentiable at a
- c. If $\lim_{x \rightarrow a} f(x)$ exists, then f is differentiable at a

d. If $\lim_{x \rightarrow 0} f(x) = \infty$, then it is
indeterminate

19. If $f(x) = (2x+1)^4$, then the fourth derivative of $f(x)$ at $x=0$ is _____

- a. 0 b. 384 c. 48 d. 240

20. Find the $\lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$?

- a. 3 b. 4 c. 5 d. 6

21. $\int \frac{x^2}{e^{x^3}} dx =$ _____

- a. $-\frac{1}{3} \ln e^{x^3} + c$ b. $\frac{-e^{-x^3}}{3} + c$ c. $\frac{1}{3} \ln e^{x^3} + c$ d. $\frac{x^3}{3e^{x^3}} + c$

22. If the second derivative of f is given by $f''(x) = 2x - \cos x$, which of the following could be $f(x)$?

- a. $\frac{x^3}{3} + \cos x - x + 1$ b. $\frac{x^3}{3} - \cos x - x + 1$ d. $x^2 + \sin x + 1$
c. $x^2 - \sin x + 1$

23. If $y = \tan x - \cot x$, then $\frac{dy}{dx} =$ _____

- a. $\sec x \csc x$ c. $\sec^2 x - \csc^2 x$
b. $\sec x - \csc x$ d. $\sec^2 x + \csc^2 x$

24. Find the gradient of the scalar field $f(x, y, z) = xy^2 - yz$.

- a. $i + (2x - z)j - yk$ c. $y^2i - zj - yk$
b. $2xyi + 2xyj + yk$ d. $(y^2i + (2xy - z)j - yk$

25. If $F(x) = f(g(x))$ where $f(-2) = 8$, $f'(-2) = 4$, $f'(5) = 3$, $g(5) = -2$ and $g'(5) = 6$, find $F'(5)$?

- a. 8 b. 24 c. 12 d. 20

26. Find the curl of $F(x, y, z) = x^2i + xyzj - zk$ at the point $(2, 1, -2)$.

- a. $2i + 2k$ b. $-2i - 2j$ c. $4i - 4j + 2k$ d. $-2i - 2k$

27. Find $k'(s)$ if $k(s) = \frac{\ln s}{s^2}$.

- a. $\frac{1}{2s^2}$ b. $\frac{-2}{s^4}$ c. $\frac{1}{s^3} + \frac{2\ln s}{s^3}$ d. $\frac{1}{s^3} - \frac{2\ln s}{s^3}$

e.

28. Given $x^4 + y^4 = 3$, find $\frac{dy}{dx}$.

- a. $\frac{-x^3}{y^3}$ b. $\frac{-x^2}{y^3}$ c. $\frac{x^3}{y^3}$ d. $\frac{x^3}{y^2}$

29. Which of the following is equal to $\int \frac{1}{\sqrt{25-x^2}} dx$?

- a. $\arcsin \frac{x}{5} + c$ b. $2\sqrt{25-x^2} + c$ c. $\frac{1}{5} \arcsin \frac{x}{5} + c$ d. $\sqrt{25-x^2} + c$

30. $\frac{d}{dx} \cos^2(x^3) =$ _____

- a. $6x^2 \sin(x^3) \cos(x^3)$ c. $-6x^2 \sin(x^3)$
 b. $6x^2 \cos(x^3)$ d. $-6x^2 \sin(x^3) \cos(x^3)$

31. Given $f(x, y) = x^2 \sin y + y^2 \cos x$, find the second partial derivative with respect to x .

b. $\frac{r!}{r!(n-r)!}$

c. $\frac{n!}{n!(r-n)!}$

d. $\frac{n!}{r!(n-r)!}$

38. Find the divergence of $G(x, y, z) = 2x^3i - 3xyj + 3x^2zk$

a. $9x^2 - 3x$

b. $6x^2 + 3x$

c. 0

d. $3x^2 - 3x$

39. Find the value of x^* that verifies the Mean Value theorem for $f(x) = x^3 + 24x - 16$ on $[0, 4]$

a. $\frac{4}{3}$

b. 2

c. $\frac{4}{\sqrt{3}}$

d. $\frac{2}{\sqrt{3}}$

40. $\int \frac{dx}{(x-1)(x+3)} = \text{_____}$

a. $\frac{1}{4} \ln \left| \frac{x-1}{x+3} \right| + c$

c. $\frac{1}{2} \ln \left| \frac{2x+2}{(x-1)(x+3)} \right| + c$

b. $\frac{1}{2} \ln |(x-1)(x+3) + c|$

d. $\frac{1}{4} \ln \left| \frac{x+3}{x-1} \right| + c$

SECTION B

Question 1

- a. Find the derivative of $f(x) = (\cos x)^x$
- b. If $v = (x^2 + y^2 + z^2)^{-1/2}$, show that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.
- c. Find $\frac{dy}{dx}$ given that
- (i) $10xy^3 - x^4y + x^3 - 7y^3 + 280 = 0$ (ii) $y = (x^2 + 1)\text{Sin}^{-1}4x$

Question 2

- a. Using the first principles, evaluate $f'(1)$ if $f(x) = \frac{5x^2 - x + 7}{x^2 + x + 10}$
- b. Find the derivative of $f(x) = \sqrt{4 - \sqrt{4 + x}}$ using the Chain Rule.
- c. Evaluate the following integrals
- (i) $\int \frac{dx}{\text{Cos } 2x + 2\text{Sin}^2 x}$ (ii) $\int xe^{2\ln x} dx$

Question 3

- a. Evaluate the integral $\int \frac{6 \cos^2 x + 2 \cos x - 23}{(\cos x - 1)^2 [5 - (1 - \cos^2 x)]} \sin x dx$
- b. Use L'hospital's rule to determine $\lim_{x \rightarrow \infty} \left[\frac{x}{\ln(1 + 2e^x)} \right]$.
- c. Given $\sin(xyz) = x + 3z + y$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

Examiners: **Monica V. Crankson / Dr Mrs C. C. Nyarko**